



ON THE THEORY OF THE DISPERSION OF MAGNETIC PERMEABILITY IN FERROMAGNETIC BODIES

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The distribution of magnetic moments in a ferromagnetic crystal is investigated. It is found that such a crystal consists of elementary layers magnetized to saturation. The width of these layers is determined. In an external magnetic field, the boundaries between these layers move; the velocity of this propagation is determined. The magnetic permeability in a periodical field parallel and perpendicular to the axis of easiest magnetization is found.

§ 1. It was pointed out by Bloch [1] and Heisenberg [2] that a ferromagnetic crystal consists magnetically of elementary regions, which are magnetized nearly to saturation. They presumed that these regions are thread-like; we shall show here that they should more likely be considered as elementary layers. This can possibly be brought into accord with the experimental evidence obtained by various authors [3] by photographing the distribution of colloidal particles of Fe_2O_3 on the surface of a ferromagnetic crystal. In nonmagnetized crystals, these elementary layers are magnetized successively in opposite directions, so that the crystal has no magnetic moment as a whole. When the crystal is magnetized, the boundaries between the oppositely magnetized layers move, so that the layers with one direction of magnetic moment grow at the cost of the layers with moments in the opposite direction.

Some authors (among them, F. Bloch [1]) tried to apply statistical considerations to determine the number and dimensions of the elementary regions in a ferromagnetic body. This is, however, quite impossible, because if there were not the demagnetizing influence of the surface of a body, for instance in an infinite body, there would be generally no elementary regions and such a body would be magnetized to saturation. This is quite analogous to the impossibility of determining the number of drops of a liquid in a condensing vapor with statistical methods, since the liquid is formed, in fact, as a continuous body. The presence of separate elementary regions magnetized in opposite directions is due only to

the demagnetizing effect of the surface, and the number and dimensions of these regions are entirely determined by the dimensions of the body.

Between two such elementary layers with oppositely directed magnetic moments, there is no discrete boundary, but there is an intermediate region, where the direction of the magnetic moments changes gradually from one direction to the opposite. We shall determine here both the distribution of the moments in such an intermediate region and the width of the elementary layers. We shall discuss the case of a ferromagnetic crystal with one distinguished axis, which is the axis of easiest magnetization, such as, for example, a single crystal of cobalt with its hexagonal axis. Every ferromagnetic body deformed in one direction (e.g., a stretched or contracted wire) is also of this kind, if the sign of a deformation is the same as the sign of a magnetostriction.

In what follows, we shall speak of a ferromagnetic crystal, but it is to be remembered that it is not necessarily a single crystal but an arbitrary body with one direction of easiest magnetization.

Such a crystal consists of layers which are parallel to the distinguished axis and magnetized to saturation parallel or antiparallel to this axis. The fact that the elementary regions in the crystal are really layers and do not have a thread-like form will be proved later on.

We shall find the distribution of the directions of magnetic moments inside the crystal in the following way. The magnetic energy of a crystal consists of two parts:

1) The energy which is due to the inhomogeneity in the distribution of the directions of magnetic moments. This energy per unit volume can be written in the form

$$1/2\alpha[(\nabla s_x)^2 + (\nabla s_y)^2 + (\nabla s_z)^2],$$

where s_x , s_y , and s_z are the components of the magnetic moment \mathbf{s} of a unit volume (its absolute value s is constant through the whole crystal and is practically equal to the saturation moment).

2) The magnetic-anisotropy energy, due to the presence of an axis of easiest magnetization. If we choose the coordinates with the z -axis along this axis, this energy per unit volume can be written as

$$1/2\beta(s_x^2 + s_y^2),$$

by showing that the minimum of energy is obtained when \mathbf{s} is directed along this axis.

Then we can find the distribution of the directions of \mathbf{s} if we demand that the energy of the crystal have a minimum, i.e.,

$$\int \{1/2\alpha[(\nabla s_x)^2 + (\nabla s_y)^2 + (\nabla s_z)^2] + 1/2\beta(s_x^2 + s_y^2)\} dV = \min, \quad (1)$$

where the integral is taken over the whole crystal. To find the distribution of moments between two layers with opposite magnetizations in the crystal, we can neglect the effects due to the surface of the crystal. These effects are fundamental for determining the width of the layers, but they can be neglected, when we wish to find only the distribution of magnetic moments in the intermediate region between two layers inside the crystal. This means that if we choose the x -axis of our coordinate system perpendicular to the layers, the distribution of magnetic moments will be independent of the coordinates y and z , the direction of \mathbf{s} changing only with x , namely from a certain direction along the z -axis in one layer through the intermediate region to the opposite direction in the next one. Relation (1) can now be written as

$$\int [1/2\alpha s'^2 + 1/2\beta(s_x^2 + s_y^2)] dV = \min, \quad (2)$$

where the prime denotes the differentiation with respect to x . The theory is such as if we assumed that the crystal is infinite. But it is necessary to remember that, in an infinite crystal, all the moments will have really the same direction, and the crystal will be spontaneously magnetized so that no layers will be present. The presence of the layers is due to the finite dimensions of the crystal; we now assume that the crystal is infinite only to find the distribution of the moments in the intermediate regions far from the surface of the crystal; the surface effects will be discussed in § 2.

In such a model, all magnetic moments are distributed in the yz -plane. We denote the angle between the \mathbf{s} and z -axis by θ . Then the components of \mathbf{s} are

$$s_x = 0, \quad s_y = s \sin \theta, \quad s_z = s \cos \theta, \quad (3)$$

and (1) takes the form (θ is a function of x only):¹

$$\int (1/2\alpha s^2 \theta'^2 + 1/2\beta s^2 \sin^2 \theta) dx = \min. \quad (4)$$

To find θ which makes this integral minimal, we write the Euler equation

$$\alpha \theta'' - \beta \sin \theta \cos \theta = 0, \quad (5)$$

whence

$$\theta'^2 - \frac{\beta}{\alpha} \sin^2 \theta = \text{const.} \quad (6)$$

The width of the layers is large as compared with the width of the intermediate region. Therefore, as the boundary condition to the equation which determines θ , we can take the conditions

$$\theta = 0 \text{ for } x = -\infty; \quad \theta = \pi \text{ for } x = +\infty$$

and

$$\theta' = 0 \text{ for } x \pm \infty \text{ or for } \theta = 0, \pi, \quad (7)$$

showing that the directions of \mathbf{s} are opposite in two adjacent layers. We see then that the constant in (6) is equal to zero, and we obtain

$$\theta'^2 = \frac{\beta}{\alpha} \sin^2 \theta. \quad (8)$$

Integrating this equation, we find the solution satisfying (7) in the form

$$\cos \theta = -\operatorname{th} \sqrt{\beta/\alpha} x. \quad (9)$$

This gives the distribution of the directions of \mathbf{s} between the two layers.

The numerical value of the constant α can be obtained approximately in the following way. The energy $1/2\alpha s'^2$ has its maximal possible value, when \mathbf{s} changes its direction after every distance equal to the lattice constant a of the crystal, i.e. when $s'^2 \cong s^2/a^2$. This maximum must be of the order kT_c if T_c is the Curie temperature (k is Boltzmann's constant). Thus, we find approximately

$$\alpha = kT_c/as^2. \quad (10)$$

The "width" of the intermediate region can be defined according to (9) as $\sqrt{\alpha/\beta}$. With (10), we obtain

$$\sqrt{\alpha/\beta} = \sqrt{kT_c/\alpha\beta s^2}.$$

¹A somewhat analogous calculation was made by Bloch [1] but from a different point of view.

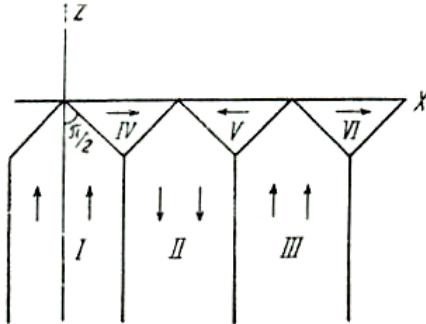


Fig. 1

For Ni, the Curie temperature is $T_c = 630$ K, the saturation moment of a unit volume $s = 480$ (for 18°C) and the lattice constant $a = 3.5 \times 10^{-8}$ cm. We took the anisotropy energy constant β from the experiments of R. Becker and M. Kersten [4]. They measured the magnetic susceptibility of stretched nickel wires and found the minimal value attained for a large strain to be 0.6. This corresponds to the constant $\beta = 1/0.6 = 1.7$. With this value, we get 2.5×10^{-6} for the "width" of the intermediate region, i.e. ca. 70 lattice constants.

§ 2. The analysis of the preceding paragraph gives only the distribution of the directions of magnetic moments in the intermediate regions, but gives nothing for determining the width of the layers. To find it, it is necessary to discuss the properties of the surface of the crystal. For this purpose, we employ the following method. We shall find the distribution of magnetic moments near the surface of the crystal for a given value d of the width of the layer, and then we shall determine this width so that the energy of the whole crystal will be a minimum.

Near the surface, there is a magnetic field; let \mathbf{H} be the macroscopical field strength. Inside the crystal, the field \mathbf{H} and the magnetic moment must satisfy the equation

$$\operatorname{div}(\mathbf{H} + 4\pi\mathbf{s}) = 0.$$

Outside it, we have

$$\operatorname{div}\mathbf{H} = 0.$$

The intermediate regions between the layers and hence the energy $1/2\alpha s^2$ are of no importance for the distribution of \mathbf{s} near the surface. If the magnetic-anisotropy energy were equal to zero, i.e. $\beta = 0$, the equilibrium, i.e. the distribution with the minimum of magnetic energy, would be such that \mathbf{H} would be zero. Then, for \mathbf{s} inside the crystal, we can write

$$\operatorname{div}\mathbf{s} = 0 \quad (11)$$

with the boundary condition

$$s_n = 0 \quad (12)$$

on the surface of the crystal if s_n is the component of \mathbf{s} normal to the surface. If β is small, we may assume that these equations are not changed appreciably, and we shall consider (11) and (12) as the equation determining the distribution of \mathbf{s} near the surface.

Let us regard, at first, the surface of a crystal (of the same kind of symmetry as in § 1) which is perpendicular to the axis of easiest magnetization, the crystal being a parallelepiped. We introduce, as in § 1, the coordinate axes with the z -axis parallel to the distinguished axis, and the yz -plane parallel to the magnetic layers of the crystal. The distribution of \mathbf{s} is uniform in the direction of the y -axis, changing with x and towards the surface with z . The intermediate regions between the layers are of no importance for the distribution of \mathbf{s} near the surface, as was already noted. We can therefore assume that \mathbf{s} is everywhere situated in the zx -plane; we denote the angle between \mathbf{s} and z -axis by φ , so that we now have

$$s_x = s \sin \varphi, \quad s_y = 0, \quad s_z = s \cos \varphi. \quad (13)$$

At a great distance from the surface, there are regular layers, say of the width d , with opposite directions of \mathbf{s} , i.e. φ is equal successively to zero or π , jumping from one of these values to another one periodically along the x -axis with period d . The distribution of \mathbf{s} is then given by Eq. (11) with the boundary conditions (if the surface of the crystal is the xy -plane) $\varphi = \pm\pi/2$ for $z = 0$ and for $z = -\infty$, φ as a function of x must change from 0 to π after every interval d .

The solution of (11), which satisfies these conditions, can be constructed in the way shown in Fig. 1. This figure shows the distribution of the magnetic moments in the crystal in the xz -plane or some other plane parallel to this one (the arrows show the direction of \mathbf{s}). In regions I and III, the moments are directed along the z -axis ($\varphi = 0$); in II along the same axis but in the opposite direction ($\varphi = \pi$); in IV, V, VI, the moments are parallel to the surface, i.e. $\varphi = \pm\pi/2$. This solution evidently satisfies the boundary conditions. It satisfies also Eq. (11) since \mathbf{s} is constant in each of the regions I, II, etc. Moreover, on the boundaries between these regions, the surface divergence is equal to zero, because the components of \mathbf{s} normal to these boundaries are everywhere equal on both sides of the boundary.

It is, of course, also possible to construct some other solution of (11); for instance, if we introduce

the same distribution as in Fig. 1 near the surface somewhere inside the crystal. But our solution is the one which gives, for a given d , the least energy of the crystal and is therefore the only one allowed by physical considerations.

We can now calculate the energy of the crystal. The energy which is due to the distribution of magnetic moments near the surface of the crystal is the anisotropy energy $\beta s_x^2/2$ (for $s_y = 0$) per unit volume. In the regions such as IV, V, VI (Fig. 1), s_x is equal to $\pm s$, i.e. the energy per unit volume is $\beta s^2/2$. Let l_1, l_2, l be the dimensions of the crystal in the directions of the x, y, z -axes, respectively. The volume of one of the regions, like IV, V, VI (Fig. 1), is then equal to $d^2 l_2/4$. There are l_1/d such regions on each of the two opposite surfaces of the crystal, their total volume is $d^2 l_1 l_2/2$, and therefore the energy due to the surface of the crystal is

$$E_e = 1/4 \beta d l_1 l_2 s^2. \quad (14)$$

In the inner parts of the crystal, the energy is due to the presence of intermediate regions between the layers. We can calculate it with the help of the results obtained in § 1. To find this energy, we consider one of such regions. Its energy is, according to § 1,

$$\int (1/2\alpha s^2 \theta'^2 + 1/2\beta s^2 \sin^2 \theta) dV,$$

where we must insert θ from (9). θ is a function of only x . Therefore, the integration over $dy dz$ gives the surface ll_2 of the layer, and we get the energy for one intermediate region as

$$\begin{aligned} & \frac{1}{2ll_2} s^2 \int_{-\infty}^{+\infty} (\alpha \theta'^2 + \beta \sin^2 \theta) dx = \\ & = ll_2 s^2 \beta \int_{-\infty}^{+\infty} \frac{dx}{\text{ch}^2 \sqrt{\beta/\alpha x}} = 2s^2 ll_2 \sqrt{\alpha/\beta}. \end{aligned}$$

There are l_1/d such regions in the crystal, and the whole internal energy of the crystal is

$$E_i = 2s^2 ll_1 l_2 \sqrt{\alpha/\beta}/d. \quad (15)$$

We now find the width d of the layers from the condition that the whole energy $E = E_i + E_s$ of the crystal must take a minimum value. Hence, we find

$$d = 2\sqrt{2l} \sqrt[4]{\alpha/\beta}, \quad (16)$$

the corresponding energy being

$$E = s^2 l_1 l_2 \sqrt{2l} \sqrt[4]{\alpha\beta}. \quad (17)$$

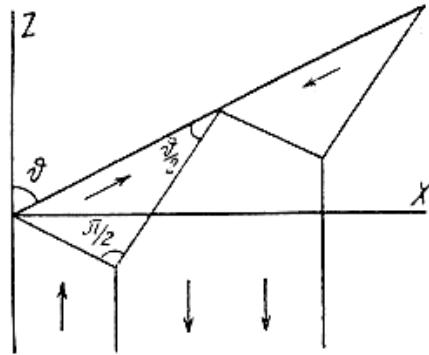


Fig. 2

If the surface of the crystal is not perpendicular to the axis of easiest magnetization, the solution of (11), which satisfies the same conditions at a large distance from the surface and with \mathbf{s} parallel to the surface on the surface of the crystal, can be constructed in the same way, and we find the distribution of \mathbf{s} in a plane parallel to xz , as it is shown in Fig. 2.

Let the angle between the z -axis and the intersection of surface of the crystal with the plane xz be ϑ_1 , and let the same angle for the opposite surface be ϑ_2 . The angle of inclination of the surface to the plane xz is of no importance for the width of the layers. In the same way as we obtained (16), we now get

$$d = 2\sqrt{l \left(\frac{1}{\sin \vartheta_1} + \frac{1}{\sin \vartheta_2} \right)^4 \sqrt{\frac{\alpha}{\beta}}}. \quad (18)$$

The length l of the crystal and the angles ϑ_1, ϑ_2 can in general change along the crystal, which is accompanied by the variation of the width of the layers.

Now it is easy to show that the crystal really consists of magnetic layers and not of threadlike regions. If the crystal consists of elementary regions of spontaneous magnetization which have the form of rectilinear prisms with the base d^2 , the distribution of the magnetic moments near the surface, satisfying (11) and (12), can be represented by Fig. 3, showing two of the elementary regions in a slit and in a plane. The energy E_i is now twice as large as (15), since the surface of the boundaries between the elementary regions is now twice as large as we had before. The surface energy

$$E_s = 1/6 \beta s^2 l_1 l_2 d,$$

as is easy to calculate. If we determine d so that the whole energy $E_i + E_s$ would be minimal, we find this minimal energy as

$$\sqrt{8/3} s^2 l_1 l_2 \sqrt{l} \sqrt[4]{\alpha\beta},$$

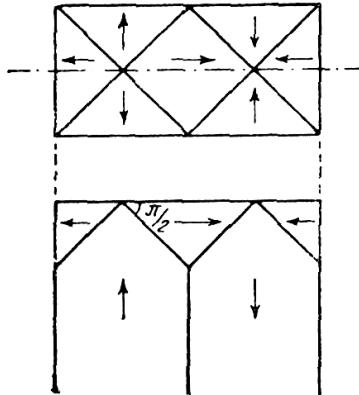


Fig. 3

i.e. it is $\sqrt{4/3}$ times larger than (17). Hence, we see that this model is energetically less advantageous than a model with layers.

Numerically, we get the width of the layers $d \cong 5 \times 10^{-3}$ or ca. 10^5 lattice constants (for $l = 1$ cm) from (16) for deformed nickel with the same parameters as in § 1.

§ 3. If the crystal is placed in an external magnetic field, which is directed parallel to the axis of easiest magnetization, the boundaries between the layers begin to move, so that the layers with magnetic moments parallel to the field become wider. We shall now determine the velocity of this propagation.

As in § 1, we consider only one intermediate region between two layers and neglect also the surface effects. The distribution of \mathbf{s} is given by (3) and (9) and is at rest when the field is absent. It begins to move with a velocity v along the x -axis when the field is introduced.

If a magnetic moment in the crystal were free, i.e. they were not under the influence of the other moments, the variation of \mathbf{s} with time would be determined by the external field. The influence of the interaction between the magnetic moments can be characterized by introducing an “effective field” in the following way.

If the macroscopical field strength inside the crystal is H , then the energy of the crystal can be written as

$$\int (1/2\alpha s'^2 - 1/2\beta s_z^2 - \mathbf{H}\mathbf{s})dV \quad (19)$$

[it is more convenient to write here the anisotropy energy as $-1/2\beta s_z^2$ instead of $1/2\beta(s_x^2 + s_y^2)$; both these expressions are evidently equivalent]. We did not write the term $\mathbf{H}\mathbf{s}$ in the formulae of § 1, because the macroscopical field strength inside the crystal was zero when there was no external field, and all \mathbf{s} were distributed in the zy -plane.

In equilibrium, this energy must be minimal, i.e. the variation of (19) with respect to \mathbf{s} must be equal to zero. This gives

$$\int (\alpha s'' + \beta s_z \mathbf{n} + \mathbf{H}) \delta \mathbf{s} dV = 0$$

(\mathbf{n} is a unit vector in the direction of the z -axis). But $\delta \mathbf{s}$ is always perpendicular to \mathbf{s} (because the absolute value of \mathbf{s} is constant); therefore, we see that

$$\mathbf{f} = \alpha s'' + \beta s_z \mathbf{n} + \mathbf{H} \quad (20)$$

must be parallel to \mathbf{s} . Hence, the quantity \mathbf{f} now plays the role of the “effective field”.

There are two kinds of the interaction between the magnetic moments in the crystal: exchange interaction and relativistic interaction. The latter is in general much weaker than the former. The exchange interaction cannot change the magnetic moment. Therefore, in the presence of the field, the magnetic moment would act as a free moment, i.e. it would rotate around \mathbf{f} , and $\dot{\mathbf{s}}$ (\cdot stands for differentiation with respect to time) satisfies the equation

$$\dot{\mathbf{s}}/\mu_0 = [\mathbf{f}\mathbf{s}]$$

with $\mu_0 = e/mc$ (and not $e/2mc$, because the moments \mathbf{s} in ferromagnetic bodies are spin moments). The approach of \mathbf{s} to \mathbf{f} is due only to the relativistic interaction. Since this interaction is weaker than the exchange interaction, we can assume that the coefficient of the term $[\mathbf{f}\mathbf{s}]$ is not altered, and we can simply add a term giving the approach of \mathbf{s} to \mathbf{f} . Thus, we come to an equation of the form

$$\dot{\mathbf{s}}/\mu_0 = [\mathbf{f}\mathbf{s}] + \lambda \left(\mathbf{f} - \frac{(\mathbf{f}\mathbf{s})\mathbf{s}}{s^2} \right). \quad (21)$$

The second term here is a vector directed from \mathbf{s} to \mathbf{f} . The constant λ is $\lambda \ll s$ in accordance with the fact that the relativistic interaction is weak. We disregard here altogether the variation of the absolute value of \mathbf{s} .

To apply this equation to the motion of the boundary between the two layers, we must determine, first, the macroscopic field \mathbf{H} inside the crystal. Since the whole distribution of \mathbf{s} is independent of the coordinates y and z , the equations determining the field, i.e.

$$\text{rot } \mathbf{H} = 0, \quad \text{div}(\mathbf{H} + 4\pi\mathbf{s}) = 0,$$

become

$$\partial H_y / \partial x = 0, \quad \partial H_z / \partial x = 0, \quad \partial(H_x + 4\pi s_x) / \partial x = 0. \quad (22)$$

If there is an external field h in the direction of the z -axis (h can be, of course, a function of time), we can put

$$H_x = -4\pi s_x, \quad H_y = 0, \quad H_z = h \quad (23)$$

if we remember that, inside the crystal far from the intermediate regions, i.e. where $s_x = 0$, the field \mathbf{H} must be equal to the external field h . If s_x were everywhere equal to zero, and no external field were present, \mathbf{H} would also be equal to zero, as in § 1.

We suppose that the field h is small (compared with $s\beta$). When the field is absent, \mathbf{s} is determined by (3) and (9). In the presence of the field, s_x is no longer equal to zero. But if h is small, s_x is also small, being proportional to h . Inserting now (20) and (23) in (21) and neglecting the terms of the second order in s_x and h everywhere, we obtain the equations

$$\begin{aligned} \dot{s}_x/\mu_0 &= \alpha(s_z s_y'' - s_y s_z'') - \beta s_y s_z - s_y h + \lambda(\alpha s_x'' - 4\pi s_x) - \\ &- \frac{\lambda}{s^2} [\alpha(s_y s_y'' + s_z s_z'') s_x + \beta s_x s_z^2], \\ \dot{s}_y/\mu_0 &= \alpha(s_x s_z'' - s_z s_x'') + (4\pi + \beta) s_x s_z + \lambda \alpha s_y'' - \\ &- \frac{\lambda}{s^2} [\alpha(s_y s_y'' + s_z s_z'') s_y + \beta s_y s_z^2 + h s_y s_z], \\ \dot{s}_z/\mu_0 &= \alpha(s_x'' s_y - s_x s_y'') - 4\pi s_x s_y + \lambda(\alpha s_z'' + \beta s_z + h) - \\ &- \frac{\lambda}{s^2} [\alpha(s_y s_y'' + s_z s_z'') s_z + \beta s_z^3 + h s_z^2] \end{aligned} \quad (24)$$

for the components of \mathbf{s} . In the absence of the field, $\dot{s}_x = \dot{s}_y = \dot{s}_z = 0$, $h = 0$, $s_x = 0$, and if we put (3) into (24), we obtain just equation (5). The quantities x_x , s_y , s_z in (24) are functions of x and the time t . We suppose that both variables enter only in a combination $x - vt$, v being the velocity of propagation of the whole distribution along the x -axis. Then $\dot{s}_x = -sv'_x$, and the same for s_y , \dot{s}_z , if the prime now denotes the differentiation with respect to $x - vt$. To solve (24), we put

$$s_x = s_x, \quad s_y = s \sin(\theta + \psi), \quad s_z = s \cos(\theta + \psi). \quad (25)$$

Here, θ is determined from (9) [and satisfies (5) or (6)], where we must introduce $(x - vt)$ instead of x . We note that ψ is small as compared with θ ; s_x and $s\psi$ are both proportional to h (and are equal to zero when $h = 0$), and we can neglect the terms of the second order in s_x and ψ . We presume, as is verified by the result, that the velocity v is also proportional to h ; therefore, we neglect

such terms as vs_x or $vs\psi$ as well. Equations (24) now become

$$\begin{aligned} &s(\alpha s\psi'' - \beta s\psi \cos 2\theta - h \sin \theta) + \\ &+ \lambda[\alpha s_x'' - (4\pi + \beta \cos 2\theta)s_x] = 0, \\ &\lambda(\alpha s\psi'' - \beta s\psi \cos 2\theta - h \sin \theta) - \\ &- s[\alpha s_x'' - (4\pi + \beta \cos 2\theta)s_x] = -\frac{sv\theta'}{\mu_0}. \end{aligned}$$

The third equation is identical with the second one. Hence, we find

$$\begin{aligned} &\alpha s\psi'' - \beta s\psi \cos 2\theta - h \sin \theta = \\ &= -\frac{sv\lambda\theta'}{\mu_0(s^2 + \lambda^2)} = -\frac{v\lambda s}{\mu_0(s^2 + \lambda^2)} \sqrt{\frac{\beta}{\alpha}} \sin \theta, \\ &\alpha s_x - (4\pi + \beta \cos 2\theta)s_x = \\ &= \frac{v\theta' s^2}{(s^2 + \lambda^2)\mu_0} = \frac{vs^2}{\mu_0(s^2 + \lambda^2)} \sqrt{\frac{\beta}{\alpha}} \sin \theta, \end{aligned} \quad (26)$$

because, according to (9),

$$\theta' = \sqrt{\beta/\alpha} \sin \theta.$$

If we introduce θ as an independent variable instead of $x - vt$, relation (26) yields

$$\begin{aligned} &\frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{d\psi}{d\theta} \right) + \left(2 - \frac{1}{\sin^2 \theta} \right) \psi = \\ &= \frac{1}{s\beta \sin \theta} \left(h - \frac{v\lambda s}{\mu_0(s^2 + \lambda^2)} \sqrt{\frac{\beta}{\alpha}} \right), \\ &\frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{ds_x}{d\theta} \right) + \left(2 - \frac{1 + 4\pi/\beta}{\sin^2 \theta} \right) s_x = \\ &= \frac{vs^2}{\mu_0\beta(s^2 + \lambda^2)} \sqrt{\frac{\beta}{\alpha}} \frac{1}{\sin \theta}. \end{aligned} \quad (27)$$

Both of these equations are of the type

$$\frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{dy}{d\theta} \right) + \left(2 - \frac{m^2}{\sin^2 \theta} \right) y = f(\theta).$$

This equation has a solution only in the cases: 1) if $m = 1, 2, \dots$ and $f(\theta)$ is zero [the solution is then $P_1^m(\cos \theta)$] or orthogonal to the solutions of the homogeneous equation; 2) if m is not an integer and $f(\theta)$ is not zero. Hence, we

see that Eqs. (27) can have a solution only if the right-hand side of the first one is equal to zero, i.e. if

$$v = \frac{\mu_0(s^2 + \lambda^2)}{\lambda s} \sqrt{\frac{\alpha}{\beta}} h. \quad (28)$$

But $\lambda \ll s$; therefore, we can write

$$v = \frac{\mu_0 s}{\lambda} \sqrt{\frac{\alpha}{\beta}} h. \quad (29)$$

This determines the velocity of motion of the boundaries between the layers in an external magnetic field directed along the axis of easiest magnetization.

If the external field h is a periodic field of the form

$$h = h_0 e^{i\omega t}, \quad (30)$$

the magnetization of the crystal is equal to zero on the average. The magnetic susceptibility is then defined as the ratio of the magnetization as a function of time to $h(t)$. For the time t , the boundary between any two layers passes a distance [we put (30) in (29) and integrate]

$$\frac{\mu_0 s}{i\lambda\omega} \sqrt{\frac{\alpha}{\beta}}.$$

There are l_1/d layers in the crystal (notations are the same as in § 2). Therefore, the total magnetization of the crystal is

$$\frac{\mu_0 s^2}{i\omega\lambda} \sqrt{\frac{\alpha}{\beta}} \frac{l_1 l_2}{d} h.$$

Hence, the magnetic susceptibility χ_l of a unit volume in the longitudinal field is

$$\chi_l = \frac{\mu_0 s^2}{i\omega\lambda d} \sqrt{\frac{\alpha}{\beta}}. \quad (31)$$

The magnetic permeability is

$$\mu_l = 1 - \frac{4\pi i\mu_0 s^2}{\omega\lambda d} \sqrt{\frac{\alpha}{\beta}}. \quad (32)$$

The expression for d can be taken from (16) or (18). It is possible then to check the dependence of d on the dimensions of the crystal, if we determine experimentally the dependence of μ_l on these dimensions.

Hence, as a function of ω, χ_l or μ_l has no proper frequencies, but a mere damping. They become infinite for $\omega = 0$ in accordance with the fact that we took into consideration no hysteresis effects. For large ω , the variation of the absolute value of \mathbf{s} (which is neglected

here) can prevail, and the formula for μ_l can become insufficient.

§ 4. We shall determine now the magnetic permeability in a transverse field, i.e. when the field h is directed along the x -axis. The effect of intermediate regions is now of no importance, and we can assume that, in the absence of the field, all the moments are directed in parallel or antiparallel (in the different layers) to the z -axis, i.e. $s_z = \pm s$ and $s_x = s_y = 0$. For the same reason, the term $\alpha s''$ in (20) is now of no importance, and we must put

$$\mathbf{f} = \mathbf{H} + \beta s_z \mathbf{n} \quad (33)$$

for the effective field. The components of \mathbf{H} are now

$$H_x = h, \quad H_y = H_z = 0. \quad (34)$$

and For the components of $\dot{\mathbf{s}}$, Eq. (21) gives

$$\begin{aligned} \dot{s}_x/\mu_0 &= -\beta s_y s_z + \lambda h - \frac{\lambda}{s^2} (hs_x + \beta s_z^2) s_x \\ \dot{s}_y/\mu_0 &= \beta s_x s_z - hs_z - \frac{\lambda}{s^2} (hs_x + \beta s_z^2) s_y \\ \dot{s}_z/\mu_0 &= hs_y + \lambda \beta s_z - \frac{\lambda}{s^2} (hs_x + \beta s_z^2) s_z, \end{aligned} \quad (35)$$

s_y and s_x are proportional to h . If we assume, as in § 3, that h is small, we can neglect the terms of the second order in s_x , s_y , and h . Instead of s_z , we can now put $\pm s$, and relation (35) yields

$$\begin{aligned} \dot{s}_x/\mu_0 &= \mp s \beta s_y + \lambda h - \lambda \beta s_x, \\ \dot{s}_y/\mu_0 &= \pm s \beta s_x \mp hs - \lambda \beta s_y. \end{aligned} \quad (36)$$

The third equation becomes an identity.

If h is determined by (30), we can solve (36) with

$$s_x = s_{x0} e^{i\omega t}, \quad s_y = s_{y0} e^{i\omega t}.$$

We then obtain

$$s_x = \frac{\mu_0^2 \beta (s^2 + \lambda^2) + i\omega \lambda \mu_0}{(i\omega + \beta \lambda \mu_0)^2 + \beta^2 s^2 \mu_0^2} h \quad (37)$$

and

$$s_y = \pm \frac{\mu_0^2 \beta s \lambda - \mu_0 s (i\omega + \mu_0 \beta \lambda)}{(i\omega + \beta \lambda \mu_0)^2 + \beta^2 s^2 \mu_0^2}. \quad (38)$$

s_y has opposite directions in different layers and therefore gives nothing for the magnetization of the whole crystal. From (37), we get the magnetic

susceptibility (for unit volume) for the magnetization in the direction of the x -axis in a transverse field:

$$\chi_t = \frac{\mu_0^2 \beta(s^2 + \lambda^2) + i\omega \lambda \mu_0}{(i\omega + \beta \lambda \mu_0)^2 + \beta^2 s^2 \mu_0^2}.$$

But $\lambda \ll s$, and we can write

$$\chi_t = \frac{\mu_0^2 \beta s^2 + i\omega \lambda \mu_0}{\beta^2 s^2 \mu_0^2 - \omega^2 + 2i\omega \beta \lambda \mu_0}. \quad (39)$$

The magnetic permeability is

$$\mu_t = 1 + 4\pi \frac{\mu_0^2 \beta s^2 + i\omega \lambda \mu_0}{\beta^2 s^2 \mu_0^2 - \omega^2 + 2i\omega \beta \lambda \mu_0}. \quad (40)$$

Hence, as a function of ω , χ_t or μ_t has a proper frequency

$$\omega_0 = \mu_0 \beta s \quad (41)$$

and the damping with a decrement

$$\gamma = \mu_0 \beta \lambda. \quad (42)$$

Evidently, $\gamma \ll \omega_0$, because $\lambda \ll s$. With the same numerical values as in § 1, we find the proper frequency for deformed nickel to be 1.5×10^{10} s⁻¹, corresponding to a wavelength of 12.6 cm. In (40) which can be also written as

$$\mu_t = 1 + 4\pi \frac{\omega_0^2 + i\omega\gamma}{\beta(\omega_0^2 - \omega^2 + 2i\omega\gamma)}, \quad (43)$$

we can distinguish some different cases according to the value of the frequency:

$$a) \quad \omega \ll \frac{\omega_0^2}{\gamma} \quad \mu_t = 1 + \frac{4\pi\omega_0^2}{\beta(\omega_0^2 - \omega^2)}, \quad (44)$$

$$b) \quad \omega - \omega_0 \sim \gamma \quad \mu_t = 1 + \frac{2\pi\omega_0}{\beta(\omega_0 - \omega + i\gamma)}, \quad (45)$$

$$c) \quad \omega \gg \omega_0 \quad \mu_t = 1 - 4\pi \frac{\omega_0^2 + i\omega\gamma}{\beta\omega^2}. \quad (46)$$

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Lev Davidovich Landau was the outstanding physicist-theorist of the XX century, Full Member of the Academy of Sciences of the USSR, the Nobel Prize winner in physics (1962) "For pioneer investigations in the theory of condensed matter and especially of liquid helium".

During 1932–1937, he was Head of the Department of Theoretical Physics at the Ukrainian Physical-Technical Institute in Kharkov (now NSC "Kharkov Institute of Physics and Technology" of NAS of Ukraine), L.D. Landau headed a chair at Kharkov State University also. Since 1937, he was Head of the Theoretical Department of the Institute for Physical Problems of the Academy of Sciences of the USSR in Moscow. He is a founder of the Kharkov's scientific school of theoretical physics.

L.D. Landau's investigations cover practically all branches of theoretical physics, ranging from fluid mechanics to quantum field theory [1]. L.D. Landau predicted the existence of neutron stars (W. Baade, F. Zwicky, L.D. Landau), he constructed a general theory of phase transitions of the second order (1936–37). The Ginzburg-Landau equation is of great importance in the theory of superconductivity. The Landau levels, Landau diamagnetism, and Landau–Lifshits equation are well known in physics. L.D. Landau constructed the theory of superfluid liquid helium-II [2] and the theory of the Fermi liquid [3–5]. L.D. Landau's investigations in plasma physics and in the theory of magnetism are of great importance. L.D. Landau formulated the theory of a two-component neutrino (independently of A. Salam, T. Lee, and C. Yang, 1957) and proposed the principle of conservation of the combined parity in the weak interactions of elementary particles [6].

A significant part in the L.D. Landau's heritage belongs to the world-famous "Course of Theoretical Physics" written jointly with E.M. Lifshits in many volumes.

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E.M. Lifshits performed investigations in various branches of condensed-matter physics, theory of gravitation and cosmology. He constructed the domain theory for ferromagnetics (1936) and deduced the equation of motion for a magnetic moment (the Landau-Lifshits equation). E.M. Lifshits contributed greatly to the theory of phase transitions of the second order [3]. He constructed a general theory of molecular interaction between solids [5]. E.M. Lifshits predicted a magnetoelectric effect (with L.D. Landau, 1941). E.M. Lifshits developed the theory of instabilities for various cosmological models of the expanding Universe [6–8].

E.M. Lifshits was the co-author (with L.D. Landau) of the world-famous “Course of Theoretical Physics” in many volumes, the first volumes of which were written in Kharkov.

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