Inducing a Magnetic Monopole with Topological Surface States

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Existence of the magnetic monopole is compatible with the fundamental laws of nature; however, this elusive particle has yet to be detected experimentally. We show theoretically that an electric charge near a topological surface state induces an image magnetic monopole charge due to the topological magneto-electric effect. The magnetic field generated by the image magnetic monopole may be experimentally measured, and the inverse square law of the field dependence can be determined quantitatively. We propose that this effect can be used to experimentally realize a gas of quantum particles carrying fractional statistics, consisting of the bound states of the electric charge and the image magnetic monopole charge.

The electromagnetic response of a conventional insulator is described by a dielectric constant \( \varepsilon \) and a magnetic permeability \( \mu \). An electric field induces an electric polarization, whereas a magnetic field induces a magnetic polarization. As both the electric field \( \mathbf{E}(x) \) and the magnetic induction \( \mathbf{B}(x) \) are well defined inside an insulator, the linear response of a conventional insulator can be fully described by the effective action \( S_0 = \frac{1}{2} \int d^4x \left( \varepsilon \mathbf{E}^2 - \frac{1}{2} \mathbf{B}^2 \right) \), where \( d^4x \) is the volume element of space and time. However, in general, another possible term is allowed in the effective action, which is quadratic in the electromagnetic field, contains the same number of derivatives of the electromagnetic potential, and is rotationally invariant; this term is given by \( S_\text{top} = \frac{\alpha}{c^4} \int d^4x \mathbf{E} \cdot \mathbf{B} \). Here, \( \alpha = \frac{c^2}{\hbar} \) (where \( c \) is Planck’s constant \( h \) divided by \( 2\pi \) and \( c \) is the speed of light) is the fine-structure constant, and \( \theta \) can be viewed as a phenomenological parameter in the sense of the effective Landau-Ginzburg theory. This term describes the magneto-electric effect \( (\theta \neq 0) \), where an electric field can induce a magnetic polarization, and a magnetic field can induce an electric polarization.

Unlike conventional terms in the Landau-Ginzburg effective actions, the integrand in \( S_\text{top} \) is a total derivative term, when \( \mathbf{E}(x) \) and \( \mathbf{B}(x) \) are expressed in terms of the electromagnetic vector potential (where \( \partial^\mu \) denotes the partial derivative; \( \mu, \nu, \rho, \tau \) denote the spacetime coordinates; \( F^{\mu\nu} \) is the electromagnetic field tensor; and \( A^\mu \) is the electromagnetic potential)

\[
\begin{align*}
S_\text{top} &= \frac{\theta}{2\pi} \int d^4x d\tau \left( \varepsilon \partial^\mu \mathbf{A} \cdot \partial^\nu \mathbf{A} \right) \\
&= \frac{\theta}{2\pi} \int d^4x d\tau \left( \varepsilon \mathbf{A} \cdot \partial^\mu \mathbf{A} \right) \\
\end{align*}
\]

Furthermore, when a periodic boundary condition is imposed in both the spatial and temporal directions, the integral of such a total derivative term is always quantized to be an integer; i.e., \( \frac{\theta}{2\pi} = \theta n \) (where \( n \) is an integer). Therefore, the partition function and all physically measurable quantities are invariant when the \( \theta \) parameter is shifted by \( 2\pi \) times an integer \( (\theta \neq 0) \). Under time-reversal symmetry, \( e^{\text{top}} \) is transformed into \( e^{\text{top}} \) (here, \( \theta = -\theta \)). Therefore, all time-reversal invariant insulators fall into two general classes, described by either \( \theta = 0 \) or \( \theta = \pi \). These two time-reversal invariant classes are disconnected, and they can only be connected continuously by time-reversal breaking perturbations. This classification of time-reversal invariant insulators in terms of the two possible values of the \( \theta \) parameter is generally valid for insulators with arbitrary interactions \( (\theta \neq 0) \). The effective action contains the complete description of the electromagnetic response of topological insulators. Topological insulators have an energy gap in the bulk, but gapless surface states protected by the effective surface symmetry. We have shown \( (\theta \neq 0) \) that such a general definition of a topological insulator reduces to the \( Z_2 \) topological insulators described in \( (4 \rightarrow 6) \) for non-interacting band insulators; this finding is a three-dimensional \( (3D) \) generalization of the quantum spin Hall insulator in two dimensions \((1 \rightarrow 10)\). For generic band insulators, the parameter \( \theta \) has a microscopic expression of the momentum space Chem-Simons form \( (3 \rightarrow 11) \). Recently, experimental evidence of the topologically nontrivial surface states has been observed in \( B_1 \ldots B_n \) alloy \((12) \), which supports the theoretical prediction that \( B_1 \ldots B_n \) is a \( Z_2 \) topological insulator \((4) \).

With periodic temporal and spatial boundary conditions, the partition function is periodic in \( \theta \) under the \( 2\pi \) shift, and the system is invariant under the time-reversal symmetry at \( \theta = 0 \) and \( \theta = \pi \). However, with open boundary conditions, the partition function is no longer periodic in \( \theta \), and time-reversal symmetry is generally broken \((\text{but only on the boundary})\), even when \( \theta = (2n + 1)\pi \). Our work in \( (3) \) gives the following physical interpretation: Time-reversal invariant topological insulators have a bulk energy gap but have gapless excitations with an odd number of Dirac cones on the surface. When the surface is coated with a thin magnetic film, time-reversal symmetry is broken, and an energy gap also opens up at the surface. In this case, the low-energy theory is completely determined by the surface term in Eq. 1. As the surface term is a Chern-Simons term, it describes the quantum Hall effect on the surface. From the general Chern-Simons-Landau-Ginzburg theory of the quantum Hall effect \( (13) \), we know that the coefficient \( \theta = (2n + 1)\pi \) gives a quantized Hall conductance of \( \sigma_H = (n + \frac{1}{2}) e^2/h \). This quantized Hall effect on the surface is the physical origin behind the topological magneto-electric (TME) effect. Under an applied electric field, a quantized Hall current is induced on the surface, which in turn generates a magnetic polarization and vice versa.

Fig. 1. Illustration of the image charge and monopole of a point-like electric charge. a. The lower-half space is occupied by a topological insulator (TI) with dielectric constant \( \varepsilon_2 \) and magnetic permeability \( \mu_2 \). The upper-half space is occupied by a topologically trivial insulator (or vacuum) with dielectric constant \( \varepsilon_1 \) and magnetic permeability \( \mu_1 \). A point electric charge \( q \) is located at \((0, 0, d)\). When seen from the lower-half space, the image electric charge \( q_1 \) and magnetic monopole \( g_2 \) are at \((0, 0, -d)\); when seen from the upper-half space, the image electric charge \( q_2 \) and magnetic monopole \( g_2 \) are at \((0, 0, d)\). The red solid lines represent the electric field lines, and blue solid lines represent magnetic field lines. (Inset) Top-down view showing the in-plane component of the electric field at the surface (red arrows) and the circulating surface current (black circles).
We propose a manifestation of the TME effect. When a charged particle is brought close to the surface of a topological insulator, a magnetic monopole charge is induced as a mirror image of the electric charge. The full set of electromagnetic field equations can be obtained from the functional variation of the action $S_0 + S_0 (14)$, and they can be presented as conventional Maxwell’s equations but with the modified constituent equations describing the TME effect (3)

$$\mathbf{D} = \mathbf{E} + 4\pi\mathbf{P} - 2\alpha P_3 \mathbf{B}$$

$$\mathbf{H} = \mathbf{B} - 4\pi\mathbf{M} + 2\alpha P_3 \mathbf{E}$$

(2)

where $P_3(x) = \theta(x)2\pi$ is the magneto-electric polarization (3), $\mathbf{D}$ is the electric displacement, $\mathbf{P}$ is the electric polarization, $\mathbf{H}$ is the magnetic field, and $\mathbf{M}$ is the magnetization. It takes the value of $P_3 = 0$ in vacuum or conventional insulators and $P_3 = \pm 1/2$ in topological insulators, with the sign determined by the direction of the surface magnetization.

Now consider the geometry as shown in Fig. 1. The lower-half space ($z < 0$) is occupied by a topological insulator with a dielectric constant $\varepsilon_3$ and a magnetic permeability $\mu_3$, whereas the upper-half space ($z > 0$) is occupied by a conventional insulator with a dielectric constant $\varepsilon_1$ and a magnetic permeability $\mu_1$. A point electric charge $q$ is located at $(0, 0, d)$ with $d > 0$. The Maxwell equations, along with the modified constituent equations and the standard boundary conditions, constitute a complete boundary value problem. To solve this problem, the method of images (15) can be used. We assume that, in the lower-half space, the electric field is given by an effective point charge $q/\varepsilon_1$ and an image charge $q_1$ at $(0, 0, d)$, whereas the magnetic field is given by an image magnetic monopole $g_1$ at $(0, 0, d)$. In the upper-half space, the electric field is given by $q/\varepsilon_1$ at $(0, 0, d)$ and an image charge $g_2$ at $(0, 0, -d)$; the magnetic field is given by an image magnetic monopole $g_2$ at $(0, 0, -d)$. The above ansatz satisfies the Maxwell equations on each side of the boundary. At the boundary $z = 0$, the solution is then matched according to the standard boundary condition, giving

$$q_1 = q_2 = \frac{1}{\varepsilon_1} (\varepsilon_1 - \varepsilon_2) (1/\mu_1 + 1/\mu_2) - 4\alpha^2 P_3^2 q$$

$$g_1 = -g_2 = \frac{4\alpha P_3}{(\varepsilon_1 + \varepsilon_2) (1/\mu_1 + 1/\mu_2) + 4\alpha^2 P_3^2} q$$

(3)

We will first take $\varepsilon_1 = \varepsilon_2 = \mu_1 = \mu_2 = 1$ below and then recover the $\varepsilon_{1,2}, \mu_{1,2}$ when discussing the experimental proposals later. The solution shows that, for an electric charge near the surface of a topological insulator, both an image magnetic monopole and an image electric charge will be induced, as compared with conventional electromagnetic media where only an electric image charge would be induced. It is notable that the magnitudes of the image magnetic monopole and image electric charge satisfy the relation $q_{1,2} = \pm (\alpha P_3) q_{1,2}$. This is just the relation $q = (2\pi \alpha) \phi$ for the electric and magnetic charges of a dyon inside the $0$ vacuum (16), with $\theta/2\pi = \pm P_3$ here.

The physical origin of the image magnetic monopole is understood by rewriting part of the Maxwell equations as

$$\nabla \times \mathbf{B} = 2\alpha P_3 \delta(z) \hat{n} \times \mathbf{E}$$

(4)

with $P_3 = \pm 1/2$ the value for the topological insulator (in the above equation, $\nabla$ is the derivative vector, $\hat{n}$ is the normal vector of the surface, and $\delta(z)$ is the Dirac $\delta$ function). The right-hand side of the above equation corresponds to a surface current density $\mathbf{j} = \sigma_{\phi}(\hat{n} \times \mathbf{E})$, which is induced by the in-plane component of the electric field and is perpendicular to this component. This current is nothing but the quantized Hall current mentioned earlier. For the problem under consideration, the surface current density is calculated as

$$\mathbf{j} = P_3 \left( \frac{e^2}{\hbar} \right) \left( \frac{q}{1 + \alpha^2 P_3^2} \right) \frac{r}{(r^2 + d^2)^2} \hat{e}_\phi$$

(5)

which is circulating around the origin (inset of Fig. 1) (here, $r$ is the radial distance and $\hat{e}_\phi$ is the tangential unit vector). Physically, this surface current is the source that induces the magnetic field. On each side of the surface, the magnetic field induced by the surface current can be viewed as the field induced by an image magnetic monopole on the opposite side.

According to the above calculation, the image magnetic monopole field indeed has the correct magnetic field dependence expected from a monopole, and it can be controlled completely through the position of the electric charge. As we started with the Maxwell’s equation, which includes $\mathbf{V} \cdot \mathbf{B} = 0$, the magnetic flux integrated over a closed surface must vanish. We can check that this is the case by considering a closed surface—for example, a sphere with radius $a$—that encloses a topological insulator. The detailed calculation is presented in the supporting online material (17). Inside the closed surface, there is not only a magnetic monopole charge, but also a line of magnetic charge density whose integral exactly cancels the point image magnetic monopole. However, when the separation between the electric charge and the surface ($d$) is much smaller than the spherical radius ($a$), the magnetic field is completely dominated by the image magnetic monopole, and the contribution due to the line of magnetic charge density is vanishingly small. Therefore, we propose here to experimentally observe the magnetic monopole in the same sense that we can experimentally observe other fractionalization, or de-confinement, phenomena in condensed matter physics. In any closed electronic system, the total charge must be quantized to be an integer. However, one can separate fractionally charged elementary

**Fig. 2.** Illustration of the experimental setup to measure the image monopole. A magnetic layer is deposited on the surface of the topological insulator, as indicated by the layer with blue arrows. (The same layer is drawn in Figs. 3 and 4.) A scanning SPM tip carries a magnetic flux $\phi$ and a charge $q$. A charged impurity is confined on the surface with charge $Q$ and distance $D$ out of the surface. By scanning over the voltage $V$ and the distance $r$ to the impurity, the effect of the image monopole magnetic field can be measured (see text).

**Fig. 3.** Illustration of the fractional statistics induced by image monopole effect. Each electron forms a dyon with its image monopole. When two electrons are exchanged, an AB phase factor is obtained (which is determined by half of the image monopole flux) and leads to statistical transmutation.
excitations arbitrarily far from each other, so that fractional charge can have well-defined meaning locally. A similar situation occurs in spin-charge separation. Whereas the total charge and the total spin of a closed system must be linked to each other, spin and charge can occur as separated local excitations. In our case, as long as $d$ is much smaller than the radius of curvature of a topological surface $a$, the local magnetic field is completely determined by a single image magnetic monopole.

Such an image monopole can be observed experimentally by a magnetic force microscope (MFM). Consider the surface of the topological insulator with a localized charged impurity (MFM). A scanning MFM tip can be applied to detect the magnetic field distribution of the image monopole. However, the charge of the impurity also generates an electric force to the tip. The contribution of the image monopole can be distinguished from other contributions by scanning both the tip position and the tip voltage. For a given position $r$, $f_{\text{min}}(r)$ defines the minimal force applied to the tip when scanning the voltage, and $D$ is the distance of the charged impurity to the surface. In the limit of $r > D$, the conventional charge interaction leads to a $1/r^6$ dependence of $f_{\text{min}}(r)$. The image monopole magnetic field leads to a dominant component

$$f_{\text{min}}(r) \approx \frac{4aP^3}{(1 + \varepsilon_2/\varepsilon_1)(1/\mu_1 + 1/\mu_2)} - 4a^2P^2 \frac{Qb}{r^3}$$

in which $Q$ is the impurity charge and $\phi$ is the net flux of the magnetic tip. For the estimated parameters $\varepsilon_2 \approx 100$ for the Bi$_{1-x}$Sb$_x$ alloy, $\varepsilon_1 = 1$, $\mu_1 = 1$, $\mu_2 = 1$, $\phi \approx 2.5\hbar c/e$, and a typical distance $r = 50$ nm, the force is $f_{\text{min}}(r) \approx 0.3$ pN/µm, which is observable in the present experiments.

In a real experimental system, the surface magnetic layer can have defects such as domains and steps that induce an inhomogeneous fringe magnetic field acting on the MFM tip. However, all surface roughness effects can only induce magnetic field of dipolar or higher order. Consequently, the contribution of surface roughness decays faster than $1/r^6$ so that the monopole contribution proportional to $1/r^3$ still dominates the long-range behavior. Further detail on the calculations is presented in the supporting online material [17].

Because the position of the image monopole is determined by that of the charge above the surface, the pure monopole is not an elementary excitation of the system, and this is different from the monopole proposed in some spin ice models [19]. However, if we consider the electrons moving on the topological surface as part of the system, the bound state of an electron and its image monopole does become a dynamical object with nontrivial properties. Such a bound state of charge and monopole is known in high-energy physics as a “dyon” (16): a composite particle with both electric and magnetic charges. Besides contributing a monopole-monopole Coulomb interaction, the image monopole also induces a statistical interaction between two dyons. When two dyons are exchanged, each of them will obtain an Aharonov-Bohm (AB) phase due to the magnetic field of the other one. The net AB phase obtained by the two-particle system during an exchange process is independent of the path of the particles on the 2D plane, which thus can be interpreted as a statistical angle of the dyon.

Fig. 4. Illustration of the experimental proposal measuring the fractional statistics of the dyons. When a gate voltage is applied to the central metallic island, the number of electrons in the central island can be tuned, which in turn changes the net flux threaded in the superconducting ring and leads to a supercurrent.
Global Cooling During the Eocene-Oligocene Climate Transition

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About 34 million years ago, Earth’s climate shifted from a relatively ice-free world to one with glacial conditions on Antarctica characterized by substantial ice sheets. How Earth’s temperature changed during this climate transition remains poorly understood, and evidence for Northern Hemisphere polar ice is controversial. Here, we report proxy records of sea surface temperatures from multiple ocean localities and show that the high-latitude temperature decrease was substantial and heterogeneous. High-latitude (45 degrees to 70 degrees in both hemispheres) temperatures before the climate transition were ~20°C and cooled an average of ~5°C. Our results, combined with ocean and ice-sheet model simulations and benthic oxygen isotope records, indicate that Northern Hemisphere glaciation was not required to accommodate the magnitude of continental ice growth during this time.

The abrupt shift to glacial conditions near the Eocene-Oligocene (E-O) boundary ~33.7 million years ago (Ma) is characterized by a ~1.5 per mil (% change in oxygen isotopic (δ18O) values of benthic foraminifera (I–3) in ~300,000 years, which is indicative of continental ice accumulation and high-latitude cooling, and an ~1-km deepening of the global calcite compensation depth (CCD) (2). Proposed causes for this fundamental change in Earth’s climate state include changes in ocean circulation due to the opening of Southern Ocean gateways (4), a decrease in atmospheric CO2 (5–8), and a minimum in solar insolation (2).

How Earth’s temperature changed during ice expansion is poorly defined, largely because benthic δ18O records do not distinguish between ice volume and temperature. Deep-sea temperature records based on foraminiferal Mg/Ca ratios show little change during ice expansion (9–11). As a result, benthic δ18O records imply E-O ice volumes that must be accommodated by Northern Hemisphere glaciation (2, 9, 12). This conclusion is nearly untenable given scant physical evidence for Northern Hemisphere ice sheets before the latest Miocene (7, 12–15). Deep-water foraminiferal Mg/Ca ratios could be affected by factors other than temperature (9, 11), including a deepening of the CCD (2) and changes in deep-water carbonate ion concentration that occurred during the E-O climate transition. Indeed, shallow-water Mg/Ca–based temperatures, from exceptionally well-preserved foraminifera deposited above the CCD, indicate ~2.5°C of cooling in the tropics (14, 15) and cast further suspicion on deep-water Mg/Ca-based temperatures across this major CCD deepening event.

Here, we report E-O sea surface temperature (SST) changes, which were determined with alkene unsaturation index (U18S) and tetraterp index (TEX486) from 11 globally dispersed ocean localities. These localities include sites 628, 803, 925, 929, 998, and 1218 in the tropics (Figs. 1 and table S1). Chronologies for these sites were previously established or refined and/or determined in this study (table S2 and fig. S1). TEX486 indices were converted to SST by use of a modified temperature calibration based on all published ocean surface sediment data (fig. S2) (20). Nonetheless, older calibrations would yield qualitatively similar results over the temperature ranges observed.

Both U18S and TEX486 SSTs show substantial high-latitude cooling between ~34 and 33 Ma (Fig. 1).