Interacting quantum walk on graphs: thermalization and entanglement

Alberto Verga[1]
R.G. Elías and K. Sellapillay

[1]Université d'Aix-Marseille, France.

Quantum walks, CIRM, January 2020

Abstract

We extend quantum walks by introducing an interaction of the particle degrees of freedom with local spins sitting on the nodes of a graph. This system allows us to investigate, in an isolated quantum system, the appearance of a thermal state and its entanglement properties. A modification of the model, in which the spins are on the network edges, exhibit a rich dynamical behavior in simple lattices, including oscillations, relaxation and localization of states.

Outline

Introduction

Node spin model: entanglement and thermalization

Edge spin model: dynamics

Conclusion

Outline

Introduction

Node spin model: entanglement and thermalization

Edge spin model: dynamics

Conclusion

Introduction

- Entanglement and thermalization of an isolated system
- Structure of the thermal state and eigenstate thermalization hypothesis
- Entanglement dynamics in spin networks induced by an itinerant particle

Outline

Introduction

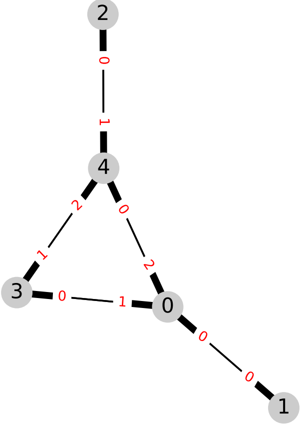
Node spin model: entanglement and thermalization

Edge spin model: dynamics

Conclusion

Node spin model: thermalization

A walker jumps between the nodes of a graph of interacting spins; the particle color interacts with the local spin favoring entanglement along graph paths



Hilbert space $|xcs\rangle = |x\rangle \otimes |c\rangle \otimes |s_0 s_1 \ldots s_{N-1}\rangle$
Coin operator C

$$\langle x'c's'| \, \mathrm{GR} \, |xcs\rangle = \left(\frac{2}{d_x} - \delta_{c,c'} \right) \delta_{x,x'} \delta_{s,s'}$$

$$\langle x'c's'| \, \mathrm{FT} \, |xcs\rangle = \frac{1}{\sqrt{d_x}} \exp(\mathrm{i} 2\pi c c'/d_x) \delta_{x,x'} \delta_{s,s'}$$

Motion operator

$$M \, |x c_y s\rangle = |y c_x s\rangle \, , \quad (x, y) \in E$$

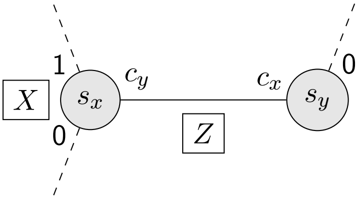
Node spin model: thermalization

Interaction operator: spin–particle (color)

$$X\,|x, 0, \dots s_x = 1 \dots\rangle = |x, 1, \dots 0 \dots\rangle$$
$$X\,|x, 1, \dots s_x = 0 \dots\rangle = |x, 0, \dots 1 \dots\rangle$$

Interaction operator: spin–spin (Ising type)

$$Z\,|xc, s = \dots s_x \dots s_y \dots\rangle = \begin{cases} -\,|xcs\rangle & \text{if } s_x = s_y = 1 \\ |xcs\rangle & \text{otherwise} \end{cases}$$



Walk operator $U = ZXMC$, $|\psi(t+1)\rangle = U\,|\psi(t)\rangle$.

Eigenstate thermalization

Isolated many–body system with hamiltonian H,

$$H|n\rangle = E_n|n\rangle$$

in a *chaotic* sate $|\psi\rangle$

The (local) observable O satisfies

$$\langle\psi|O|\psi\rangle \approx \langle n|O|n\rangle \approx \mathrm{Tr}\,\rho_{MC}O(E)$$

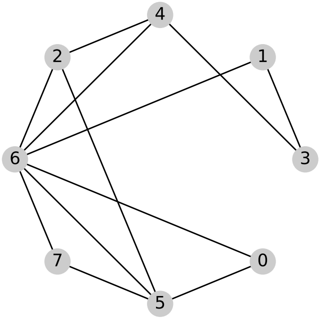
for $E \approx E_n$.

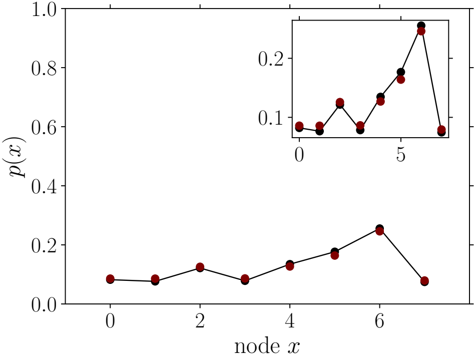
We say that $|\psi\rangle$ is a *thermal* state: most observables satisfy ETH.

Eigenstate thermalization

Position distribution in a random graph (Fourier coin):

$$\rho(t) = |\psi(t)\rangle\langle\psi(t)|\,, \quad |\psi(t)\rangle = \sum_{xcs}(t)\psi_{xcs}\,|xcs\rangle$$



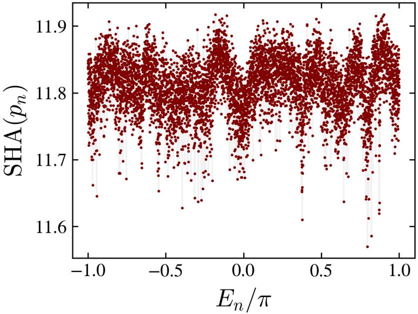


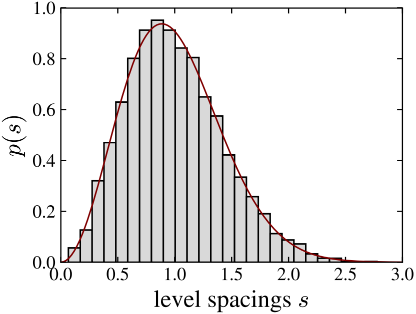
$$p(x,t) = \mathrm{Tr}_{\bar{x}}\,\rho(t) = \sum_{cs}|\psi_{xcs}(t)|^2$$

Eigenstate thermalization

Shannon entropy:

$$S(n) = -\sum_{xcs} |v_n(xcs)|^2 \log |v_n(xcs)|^2, \; v_n(xcs) = \langle xcs|n\rangle$$





Gaussian unitary ensemble distribution of eigenvalues spacing s:

$$p(s) = \frac{32s^2}{\pi^2} \mathrm{e}^{-4s^2/\pi}$$

Thermal state structure

The particle color–spin interaction favors one dimensional paths drown on the graph. We show that the von Neumann entropy

$$S_l(t) = -\operatorname{Tr}\rho_l(t)\log\rho_l(t) \leq \log D_l - \frac{D_l^2}{2D\ln(2)}$$

$(l = \{x, c, s\})$ is related with the *minimal cycle basis* entropy:

$$S_s \approx S_C = \log\left[\sum_{n=1}^{|B^\star|}\operatorname{len}(b_n^\star)\right]$$

where $b_n^\star \in B_C$ stands for the minimal length cycle basis of the graph G cycle space, B_C.

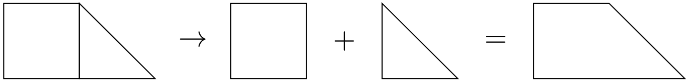
Remark: cycle basis

The cycle space B_C of a graph have a structure of vector space over the field $\{0, 1\}$; one may thus define a basis of this space whose elements b_n are cycles.

$$B = \{b_n \in B_C,\ n = 1, \ldots, |B| = |E| - |V| + 1\}$$

Among these basis sets one has minimal length:

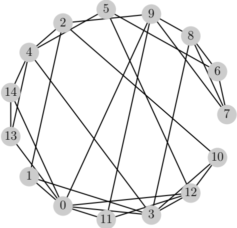
$$B^\star = \left\{ b_n^\star \mid \sum_n \operatorname{len} b_n^\star = \min_B \operatorname{len}(B) \right\}$$



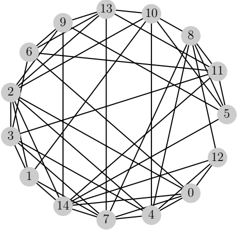
The graph on the left has a cycle basis of dimension 2, a rectangle and a triangle; a linear combination of the two basis cycles gives another cycle in the set of cycles in G.

Thermal state structure

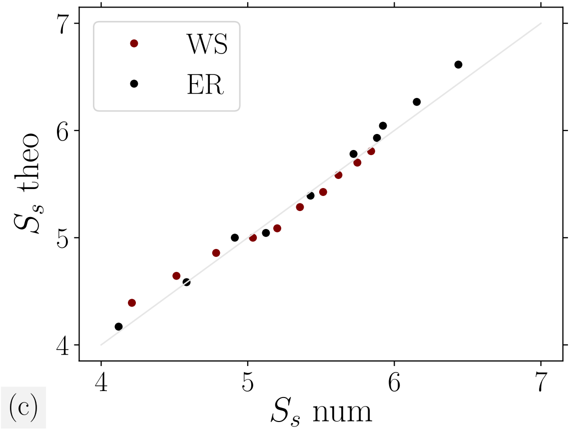
Strings of entangled spins



WS



ER



ER, WS $|V| = 5, \ldots, 15$ nodes

Outline

Introduction

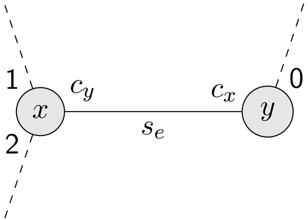
Node spin model: entanglement and thermalization

Edge spin model: dynamics

Conclusion

Edge spin model: dynamics

Hilbert space $|xcs\rangle = |x\rangle \otimes |c\rangle \otimes |s_0 \dots s_{|E|-1}\rangle \in \mathcal{H}_G$



Spin–color interaction on edge $e = (x, y)$:

$$S(J) = \exp(-\mathrm{i}H),\ H = -\frac{J}{4}\boldsymbol{\tau} \cdot \boldsymbol{\sigma}$$

Quantum walk step operator:

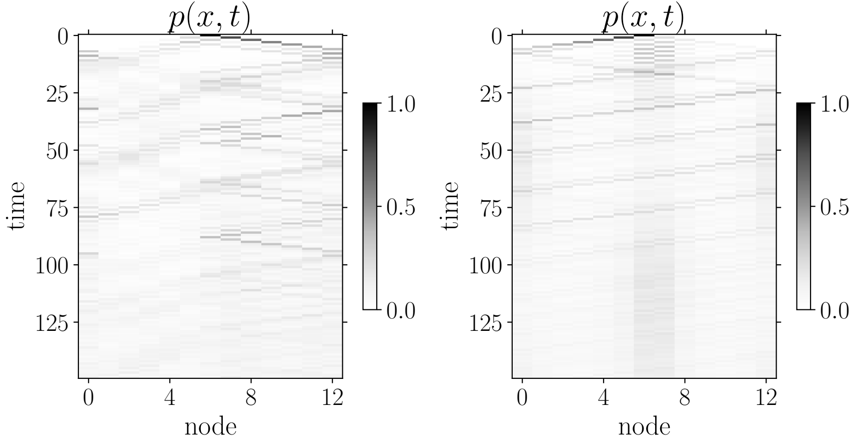
$$U = SMC$$

Topology driven edge states

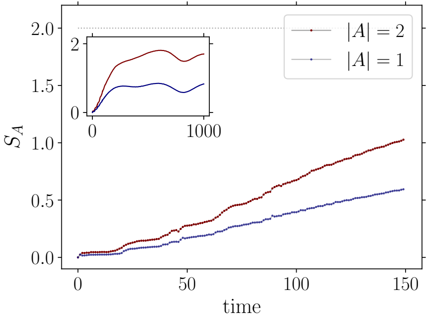
One dimensional lattice walk with a rotation coin $R(\theta) = \exp(\mathrm{i}\sigma_y\theta)$. In the free case $(J = 0)$ a change of topology arises at $\theta = \pi/2$.

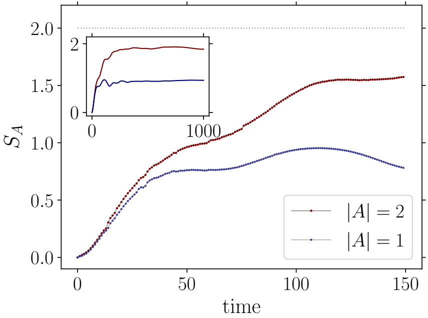
We introduce an interface at the center of the lattice separating a left and right regions with different coin angles. In the trivial case $\theta_L, \theta_R < \pi/2$ and in the topological case $\theta_L < \pi/2 < \theta_R$.

Topology driven edge states



Topology driven edge states, entanglement





Outline

Introduction

Node spin model: entanglement and thermalization

Edge spin model: dynamics

Conclusion

Conclusion

The interacting quantum walk allows us to investigate, within a simple framework, fundamental physical mechanisms arising in condensed matter.

References:

- Interacting quantum walk on a graph, A.D. Verga, Phys. Rev. E **99**, 012127 (2019)
- Thermal state entanglement entropy on a quantum graph, A.D. Verga and R.G. Elías, Phys. Rev. E **100**, 062137 (2019)
- Dynamics of an interacting quantum quantum walk on a graph, K. Sellapillay, this conference.