# Skyrmions

Alberto Verga[1]

[1]Université d'Aix-Marseille, France.

March 14, 2018

# Abstract

Skyrmions are nontrivial topological structures arising in chiral metals and ferromagnetic films. Using skyrmions as an example, we explore some elementary concepts of topological phases in condensed matter. We proceed by traveling over different scales form macroscopic to microscopic and back.

# Outline

Thermodynamics

Statistical mechanics

Quantum mechanics

Micromagnetics

Applications

# Outline

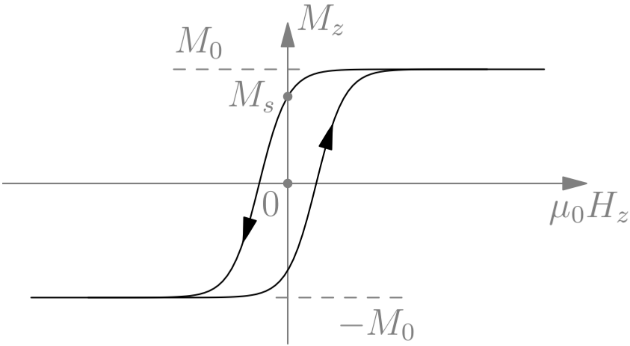
# Thermodynamics

Equation of state:

$$\boldsymbol{M} = \boldsymbol{M}(T, \boldsymbol{H})$$

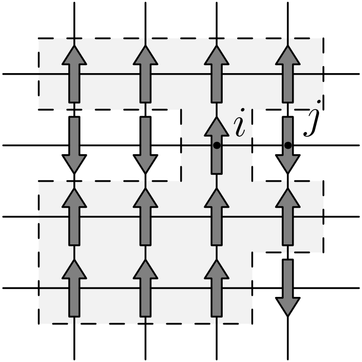
relates the magnetization $\boldsymbol{M}$ to the temperature $T$ and the applied magnetic field $\boldsymbol{H}$. A simple form is given by the implicit function:

$$m = m_0 \tanh\left(\frac{Jm}{T} + \frac{\mu_B \mu_0 H}{T}\right)$$



# Outline

# Statistical mechanics




Ferromagnetism is a quantum effect, Heisenberg hamiltonian:

$$\mathcal{H}_{\mathsf{ex}} = -J \sum_{\langle i,j \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

The free energy $F$ is computed from the partition function $Z$,

$$F(T, \boldsymbol{H}) = -T \ln Z(T, \boldsymbol{H})$$

in the thermodynamic limit.

The partition function is a sum over all microscopic energy states of the macroscopic system,

$$Z(T, \boldsymbol{H}) = \sum_{\{\boldsymbol{S}_i\}} \mathrm{e}^{-\mathcal{H}[\boldsymbol{S}_i]/T}, \quad \mathcal{H} = \mathcal{H}_{\mathsf{ex}} - \mu_B \mu_0 \sum_i \boldsymbol{H} \cdot \boldsymbol{S}_i.$$

contains the applied field term.

# Statistical mechanics

The probability of a microscopic configuration $S = \{\boldsymbol{S}_i\}$ is,

$$P(S) = \frac{\mathrm{e}^{-\mathcal{H}[S]/T}}{Z}$$

The magnetization is readily computed using the partition function:

$$\boldsymbol{M} = -\frac{1}{\mu_0 V Z} \frac{\partial}{\partial \boldsymbol{H}} Z = \frac{\mu_B}{V} \sum_i \langle \boldsymbol{S_i} \rangle$$

the macroscopic magnetization is the thermal averaging of the microscopic spins; each spin being associated with an elementary magnetic moment $\mu_B$. Introducing an effective magnetic field:

$$H_{\text{eff}} = H + \frac{J}{\mu_B \mu_0} \sum_i \langle S_i \rangle$$

one obtains the equation of state.

# Outline

# Quantum mechanics

The value of the exchange energy $J$ depends on the electronic structure of the ferromagnetic material; therefore its computation is quantum mechanical, we need to solve the Schrödinger equation to find the matrix element,

$$J = \langle \boldsymbol{x}_1 \boldsymbol{x}_2 | U | \boldsymbol{x}_2 \boldsymbol{x}_1 \rangle$$

where $U$ is the Coulomb interaction (operator) and $|\boldsymbol{x}_1 \boldsymbol{x}_2\rangle = |\boldsymbol{x}_1\rangle \otimes |\boldsymbol{x}_2\rangle$ is the position two particle state.

# Outline

# Micromagnetics

Landau free energy (thermodynamics)

$$F(M,H) = F_0(T) + a(T)M^2 + bM^4 - \mu_0 MH$$

To include inhomogeneity and temporal variations one introduces an intermediate scale magnetization (coarse-graining):

$$\boldsymbol{M}(\boldsymbol{x},t) \sim \frac{1}{V(\ell)} \sum_{i \in B_{\boldsymbol{x}}(\ell)} \langle \boldsymbol{S}_i(t) \rangle$$

The exchange free energy depends on the magnetization gradients:

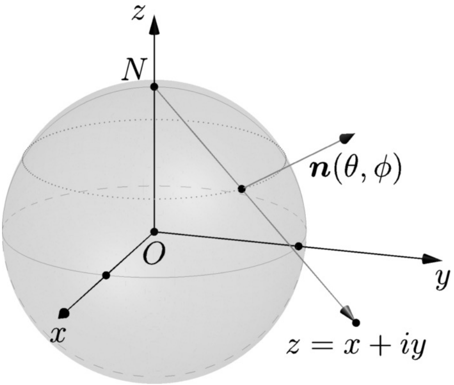
$$F_{\mathsf{ex}} = \int_V \mathrm{d}V\, A|\nabla \boldsymbol{M}|^2,$$

Landau-Lifshitz equation

$$\frac{\partial}{\partial t}\boldsymbol{m} = -\frac{\gamma}{M_s}\boldsymbol{m} \times \frac{\delta F}{\delta \boldsymbol{m}}$$

$$\frac{\partial}{\partial t}\boldsymbol{m} = \kappa \boldsymbol{m} \times \nabla^2 \boldsymbol{m}, \quad \kappa = 2\gamma A M_s$$

# Micromagnetics: stereographic projection




$$w = \frac{m_x + \mathrm{i} m_y}{1 + m_z} = \tan\frac{\theta}{2}\mathrm{e}^{\mathrm{i}\phi}\,,$$

$$m_x = \frac{w + \bar{w}}{1 + |w|^2}$$

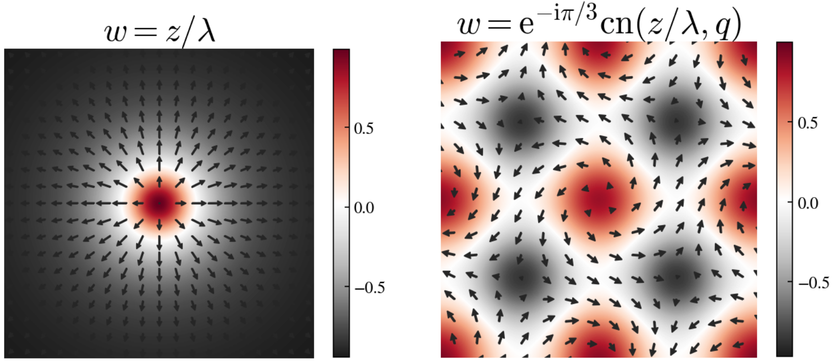
$$m_y = \frac{1}{\mathrm{i}}\frac{w - \bar{w}}{1 + |w|^2}$$

$$m_z = \frac{1 - |w|^2}{1 + |w|^2}\,,$$

Landau-Lifshitz equation in the stereographic plane:

$$\mathrm{i}\partial_t w = -\kappa\partial\bar{\partial}w + \frac{2\kappa\bar{w}}{1 + |w|^2}\partial w\bar{\partial}w,\ \Rightarrow\ \bar{\partial}w(z) = 0\,.$$

# Micromagnetics: skyrmions




Isolated skyrmion and skyrmion lattice, obtained from two different choices of $w = w(z)$, the stereographic representation of the unit magnetization vector.

# Micromagnetics: skyrmion topology

Skyrmion topological charge:

$$Q = \int \frac{\mathrm{d}\boldsymbol{x}}{4\pi} \boldsymbol{m} \cdot \partial_x \boldsymbol{m} \times \partial_y \boldsymbol{m} \,,$$

which can be put in the form:

$$Q = \frac{1}{4\pi} \int \mathrm{d}\theta(\boldsymbol{x}) \mathrm{d}\phi(\boldsymbol{x}) \sin\theta(\boldsymbol{x}) \sim \frac{\Omega}{4\pi} \in \mathbb{Z}$$

and, in the stereographic plane:

$$Q = \int \frac{\mathrm{d}x\mathrm{d}y}{\pi} \frac{|\partial w|^2 - |\bar{\partial} w|^2}{(1+|w|^2)^2} \,.$$

The skyrmion energy,

$$E = 4AM_s^2 \int \mathrm{d}x\mathrm{d}y \frac{|\partial w|^2 + |\bar{\partial} w|^2}{(1+|w|^2)^2} \,.$$

is finite,

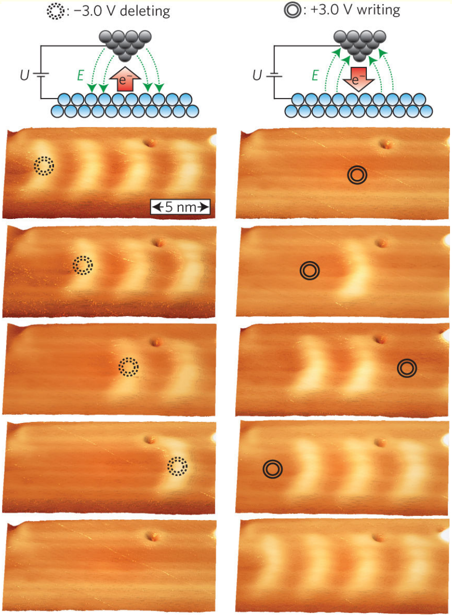
$$E = 4\pi AM_s^2 Q \,,$$

# Outline

# Applications

Nanometric magnetic structures like skyrmions are good candidates to create ultradense memories (Fert et al. 2013), and other spintronic devices (Rosch, 2016). To achieve such goal it is important to find physical effects allowing the manipulation of skyrmions to control their nucleation, destruction and motion. One possibility is to interact with the skyrmion through a spin polarized current: the itinerant spins can exerce a torque on the fixed magnetization, and hence change their orientation. This effect was discovered by Slonczewski in 1996 and is called the spin transfer torque.

# Applications: deleting and writing skyrmions




Triple layer of Fe on Ir(111); a stable skyrmion state is obtained in a 2.7 T magnetic field; skyrmions are manipulated with electric fields using its influence on the value of the exchange constant $J$ (Hsu et al. 2016).

# Applications: future

Another possibility is to build heterostructures with a layer of a topological insulator (Moore 2010) in contact with a magnetic film, to manipulate the skyrmion using electric fields. A topological insulator is a bulk insulator that supports spin polarized conducting channels at their surface; this effect is related to the topology of the electronic bands, which in a topological insulator are inverted (the valence band is above the conducting one, separated by a large gap), combined with a strong spin-orbit coupling. In these heterostructures, one can use the magnetoelectric effect (Qi 2009), intrinsic in three dimensional topological insulators, to interact with the skyrmion. Under the influence of the helical electronic modes of the surface polarized currents, a charge is induced in the skyrmion, which then can be displaced by an electric field.