Photon quantum mechanics and beam splitters

C. H. Holbrow,[a] E. Galvez, and M. E. Parks
Department of Physics and Astronomy, Colgate University, Hamilton, New York 13346

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We are developing materials for classroom teaching about the quantum behavior of photons in beam splitters as part of a project to create five experiments that use correlated photons to exhibit nonclassical quantum effects vividly and directly. Pedagogical support of student understanding of these experiments requires modification of the usual quantum mechanics course in ways that are illustrated by the treatment of the beam splitter presented here. © *2002 American Association of Physics Teachers*.

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I. INTRODUCTION

By using recent advances in the production and detection of correlated photons, we are developing a sequence of five experiments for undergraduates to demonstrate important aspects of quantum superposition. Doing these experiments will prepare students to understand issues of quantum cryptography, quantum computing, and quantum teleportation. More directly, the experiments will show that the photon exists, that a photon interferes with itself, that small modifications of the observing apparatus can erase and restore interference, and that there exist other kinds of photon interference than the one-photon sort. The apparatus will also be used to exhibit the nonlocal nature of quantum mechanics by showing the violation of Bell's inequalities. Chiao, Kwiat, and Steinberg have written an excellent review of these and related experiments.[1] More information is also available on our website.[2]

Because we think that these experiments will make the unusual and unintuitive features of quantum mechanics tangible and vivid, we want to include them in our introductory quantum mechanics course (most recently taught from Griffiths' text[3]). However, for the experiments to be meaningful, we will need to expand the material on state vectors, study extensively the two-state system, go more deeply into the linear algebra needed to describe transformations of state vectors by experimental apparatus, and show how superposition leads not just to conventional interference but also to entanglement and interference of correlated pairs. Room for this additional material will probably be at the expense of wave functions and the hydrogen atom. Hence, the successful incorporation of our experiments into undergraduate quantum mechanics implies a substantial revision of the usual syllabus.

This paper analyzes a beam splitter to show the kind of change we have in mind for our syllabus. We omit discussion of the quantum eraser[1] and the violation of Bell's inequalities[4,5] even though the last experiment will be the climax of our program. We concentrate on three experiments for which the beam splitter is a key part. We show how to use linear algebra to describe the operation of a beam splitter, an approach that we believe will exercise students in the use of mathematical tools important for the analysis of quantum state vectors. We show how these tools can be used to analyze a more complicated experimental apparatus, the Mach–Zehnder interferometer, as a combination of beam splitters. We then apply the same tools of analysis to predict the completely nonclassical output that can be produced with two-photon states which can be made with our apparatus.

II. BEAM SPLITTER EXPERIMENTS

A. Underlying technology needs

All of the experiments require correlated pairs of photons as their inputs. The possibility of doing any of our planned experiments depends on the capability to generate, select, and detect such photon pairs. Two devices, the down converter and the avalanche photodiode, provide the technology that makes our experiments feasible.

Correlated pairs can be created with a down converter, a crystal specially cut to exploit its birefringence and its nonlinear properties so that a single entering photon, in the deep blue or ultraviolet, results in the output of two lower energy photons. The conversion efficiency of 10^{-12}–10^{-10} is very low. Therefore, it is necessary to use a laser that can supply enough input photons to compensate for this low efficiency and also for inefficiencies in the selection of special pairs from among those generated. A laser with a power of the order of 100 mW is needed. (At 450 nm, 100 mW corresponds to 2.3×10^{17} photons/s.) The outgoing two photons are correlated in time because they are produced at the same instant; they are correlated in momentum and in frequency by the conservation laws; and with proper choice of crystal, crystal orientation, and spatial filters, their polarization states can be correlated. Any of these correlation properties can be the basis of coincidence experiments that dramatically exhibit the quantum nature of light.

Even with 100 mW there are no photon pairs to waste, and it is necessary to detect coincidences with as much efficiency as possible. Avalanche photodiodes are extremely sensitive detectors that can register single photons with efficiencies as high as 80%. Their use makes it possible to measure coincidences of correlated pairs and accumulate statistically significant numbers of counts within the duration of a typical undergraduate laboratory.[6]

B. The experiments: The photon exists

The layout of the experiment to show directly the existence of the photon is illustrated schematically in Fig. 1. Two photons emerge simultaneously from the down converter. One goes to detector D_0; the other goes to the beam splitter BS and then to detector D_1 or D_2. If the photon were a classical wave, it would split at the beam splitter, and some

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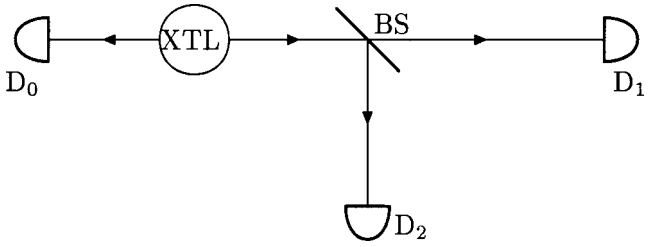

Fig. 1. Two photons emerge from XTL, the down converter—the incident laser beam that excites the down converter is not shown. One photon goes to detector D_0; the other goes by way of the beam splitter BS to D_1 or D_2. The absence of coincidences between detectors D_1 and D_2 demonstrates the existence of the photon.

amplitude would appear in both D_1 and D_2. The experiment consists of looking at coincidences D_0-D_1, D_0-D_2, and D_1-D_2. The observation is that D_0-D_1 or D_0-D_2 will occur, but never D_1-D_2.[7] The absence of D_1-D_2 coincidences is what we mean when we say the photon exists. Clearly the beam splitter is the heart of the apparatus here.

C. The experiments: The photon interferes with itself

The next experiment uses a Mach–Zehnder interferometer as shown in Fig. 2. Changing the length of one arm, say by moving mirror M_1, will change the phase relationship between the two paths from the input to the beam splitter BS_2. As the phase is changed, the count rate in detector D_1 will vary back and forth between some maximum and nearly zero, corresponding to variations from constructive to destructive interference. Such variations correspond to the appearance and disappearance of interference fringes as in a Michelson interferometer; therefore, in what follows we will use the word "fringes" to refer to these variations in count rate.

To show that a single photon interferes with itself, that is "takes both paths," look at D_0-D_1 coincidences as a function of the change in phase. This arrangement guarantees that the observed interference fringes have built up from counts occurring when there is only one photon in the interferometer. Similar fringes occur in detector D_2 but 90° out of phase with those in D_1.

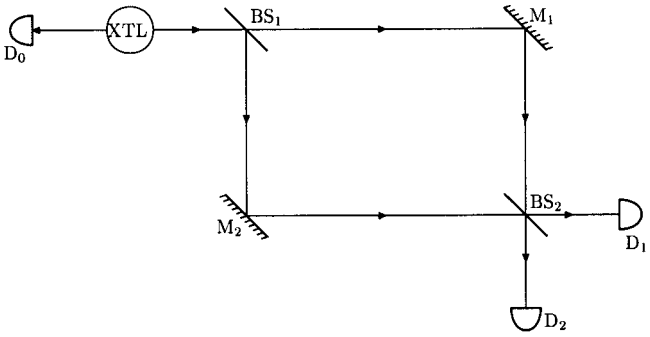

Fig. 2. The motion of the mirror M_1 introduces a phase change in one arm of the Mach–Zehnder interferometer. Interference fringes appear in the counts in D_1 and D_2. Detection in coincidence with D_0 assures that there is only one photon in the interferometer at the time of detection and, therefore, that a photon interferes with itself.

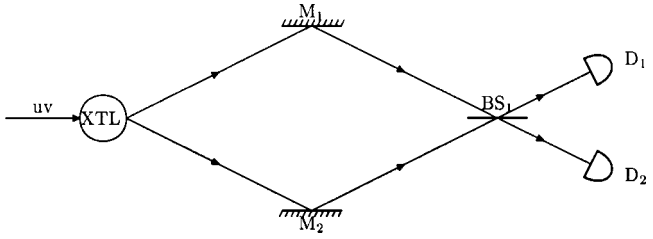

Fig. 3. When two photons from a down converter are inputs to a beam splitter, there can be interference fringes in the coincidences, even though the counts in each counter show no evidence of interference.

Note that the Mach–Zehnder interferometer consists of two mirrors and two beam splitters, so that once again beam splitters are the heart of the apparatus.

D. The experiments: Two-photon interference

The quantum nature of light is made vividly apparent when the apparatus is arranged so that both the correlated photons from the down converter simultaneously enter the inputs of the beam splitter as shown schematically in Fig. 3. Quantum mechanics predicts that in such a case the two photons will always arrive at the same output of the beam splitter, either both at D_1 or both at D_2, but never one photon at D_1 and the other at D_2.

Moreover, when a phase difference is introduced between the two arms of the apparatus, interference fringes will occur in the D_1-D_2 coincidence counts, even though they are not observed in the counts observed at D_1 or at D_2 alone.

III. WHAT DO STUDENTS NEED TO KNOW TO UNDERSTAND THE EXPERIMENTS?

A. Background

Our syllabus is intended for students who have had a three-term introductory physics course plus a one-term course called "Waves and Modern Physics." The waves course introduces students to the solution of the harmonic oscillator equation, the representation of waves by complex exponentials, Fourier analysis, and to the one-dimensional Schrödinger equation applied to the particle in a box or to piecewise continuous potentials. The mathematics background of the students is three semesters of calculus; a few have had a linear algebra course devoted largely to basic properties of matrices and their manipulation. Very few if any students have had a course in differential equations.

B. Matrix mechanics

Students need a good grounding in the quantum mechanics of two-state systems in order to understand these experiments. As part of this grounding, we will spend more time than is customary on state vectors and matrix operators. We will also examine closely systems of identical particles, especially entangled two-particle states that are central to our project.

Our plan is to introduce two-state systems using the polarization states of photons as treated by French and Taylor[8] and the somewhat more complete treatment of Lipkin.[9] We will then treat the case of the spin 1/2 particle, adapting Feynman's Stern–Gerlach analysis[10] and going on to a treatment such as Griffiths'.[3] Feynman has discussed a rich variety of

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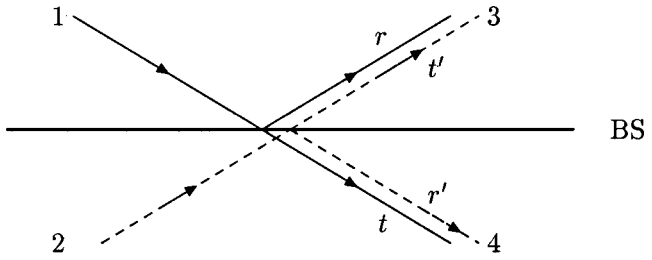

Fig. 4. A schematic representation of an ideal beam splitter with input ports 1 and 2 and output ports 3 and 4. The amplitudes for reflection and transmission from port 1 are, respectively, r and t. The corresponding amplitudes for a photon entering 2 are r' and t'.

systems that can be described using 2×2 matrices, and we will use some of these to familiarize students with the terminology, properties, and use of matrices. We also want to teach students how to interpret quantum situations in terms of probability amplitudes in the style of Feynman.[11,12]

We will emphasize the fundamental ideas that any dynamical variable has a matrix representation, the eigenvalues of that matrix are the only possible results of measurements of the variable, and that any state of the system can be represented as a linear combination of eigenstates. We will need to teach how to find the eigenvalues and eigenvectors of a matrix. We must also teach the idea of representing a state vector in terms of a basis, the idea of a complete set of states, how to transform from one basis to another, why the matrices of observables must be Hermitian adjoint, and what unitary transformations are and why they are important.

IV. PHOTONS ON A BEAM SPLITTER AS A TWO-STATE SYSTEM

Figure 4 represents an idealized model of a beam splitter. It has two input ports and two output ports. We assume it to be lossless. For photons incident through port 1, we denote the reflection and transmission amplitudes as r and t, respectively. For photons incident through port 2, these amplitudes are r' and t'. We simplify the analysis by assuming that these fractions are independent of the photons' angle of incidence or their state of polarization.

A. Unitarity and the beam splitter matrix

The main point here is to recognize that the two possible inputs of a photon into a beam splitter can be represented as a two-state system. One possible input state $|\text{in}\rangle$ is a photon entering port 1; another is a photon entering port 2. Think of these as basis vectors $\binom{1}{0}$ and $\binom{0}{1}$, respectively. Similarly, we choose an exit representation for the output states in which $\binom{1}{0}$ represents a photon exiting port 3 and $\binom{0}{1}$ represents one exiting 4.

Described in these terms, the beam splitter performs a linear transformation $\mathbf{R}$ that converts $|\text{in}\rangle$ to $|\text{out}\rangle$, that is, $|\text{out}\rangle=\mathbf{R}|\text{in}\rangle$. To find $\mathbf{R}$ in this representation, note that when the reflection and transmission amplitudes are r and t, respectively, for an input state of $\binom{1}{0}$, the output state will be $\binom{r}{t}$ in the exit representation. A similar argument for r' and t' and an input state of $\binom{0}{1}$ yields

$$\mathbf{R}=\begin{pmatrix} r & t' \\ t & r' \end{pmatrix}.$$

Because of the conservation of probability, the matrix $\mathbf{R}$ must be unitary. This condition means that the Hermitian adjoint $\mathbf{R}^{\dagger}$ equals the inverse $\mathbf{R}^{-1}$, a fact that yields useful relationships among the elements of the matrix of $\mathbf{R}$. The inverse of $\mathbf{R}$, like that of any matrix, is the transpose of its cofactor matrix divided by its determinant. Therefore,

$$\frac{1}{rr'-tt'}\begin{pmatrix} r' & -t' \\ -t & r \end{pmatrix}=\begin{pmatrix} r^* & t^* \\ t'^* & r'^* \end{pmatrix}. \tag{1}$$

The determinant of a unitary matrix has a modulus of one, so $rr'-tt'=e^{i\gamma}$. Because the factor $e^{i\gamma}$ multiplies every element of the matrix, it will not affect relative phases between terms, and it may be assigned a convenient value. If we choose $\gamma=0$, the factor $rr'-tt'$ is just 1. By equating the corresponding elements of the right- and left-hand sides of Eq. (1), we obtain

$$r=r'^* \tag{2a}$$

$$t=-t'^*. \tag{2b}$$

If we rewrite these factors as complex exponentials, $|r|e^{i\delta_r}$, $|r'|e^{i\delta_{r'}}$, $|t|e^{i\delta_t}$, and $|t'|e^{i\delta_{t'}}$ and divide Eq. (2b) by Eq. (2a), we obtain

$$\frac{|t|}{|r|}e^{i(\delta_t-\delta_r)}=-\frac{|t'|}{|r'|}e^{-i(\delta_{t'}-\delta_{r'})}. \tag{3}$$

Equations (2) show that $|t|=|t'|$ and $|r|=|r'|$, so these terms may be canceled in Eq. (3). Then dividing Eq. (3) by the right-hand side and using the fact that $-1=e^{i\pi}$, we obtain

$$\delta_t-\delta_r+\delta_{t'}-\delta_{r'}=\pi, \tag{4}$$

as Zeilinger has shown.[13]

For the common case of a beam splitter that has the same effect on a beam incident through port 1 as on a beam incident through port 2, that is, a symmetric beam splitter, $r=r'$, $t=t'$, and $\delta_t-\delta_r=\delta_{t'}-\delta_{r'}=\pi/2$, and the transmitted wave leads the reflected wave in phase by $\pi/2$ rad. This phase difference introduces an important factor of i into the transmission amplitudes, a factor that is usually introduced with no more explanation than "unitarity implies" it.

The 50–50 symmetric beam splitter is particularly simple. For this case not only do we have $r=r'$ and $t=t'$, but now $|r|=|t|=|r'|=|t'|$. It then follows from Eq. (2) that r must be real and t must be pure imaginary. Given that t leads r in phase by $\pi/2$ rad and that the determinant $rr'-tt'=1$, it follows that $r=1/\sqrt{2}$ and $t=i/\sqrt{2}$. The matrix $\mathbf{R}$ is then[14]

$$\mathbf{R}=\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}. \tag{5}$$

B. Application to a Mach–Zehnder interferometer

As Fig. 2 shows, a Mach–Zehnder interferometer consists essentially of two beam splitters. The mirrors can be ignored because their effects in their respective arms balance out. The output is just two applications of $\mathbf{R}$: $|\text{out}\rangle=\mathbf{RR}|\text{in}\rangle$. It is made to function as an interferometer by inserting a phase shifter into one arm. For example in Fig. 2, the phase shifter might be just the motion of the mirror M_1. In our output representation, a phase shifter can be represented as

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$$\Phi = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix}. \tag{6}$$

Then a Mach–Zehnder interferometer is

$$|\text{out}\rangle = \mathbf{R}\Phi\mathbf{R}|\text{in}\rangle = \frac{1}{2}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
$$= ie^{i\phi/2}\begin{pmatrix} \sin\dfrac{\phi}{2} \\ \cos\dfrac{\phi}{2} \end{pmatrix},$$

from which it follows that the probabilities for being in exit ports 3 and 4 are

$$\langle \text{out}|\text{out}\rangle = \begin{pmatrix} \sin^2\left(\dfrac{\phi}{2}\right) \\ \cos^2\left(\dfrac{\phi}{2}\right) \end{pmatrix}. \tag{7}$$

Interference fringes corresponding to these probabilities occur as ϕ is varied.

C. Two-photon interference

As noted above, our experimental setup is designed to introduce two photons into the apparatus at the same time. We consider the case for which the incident photons are in identical polarization states. There then result two-photon interference effects that can only be explained by quantum mechanics.

To describe the states of a two-photon system, we will draw on the students' earlier introduction to composite systems. They will have seen that the state of an assembly of noninteracting particles can be described as the product of the states of the individual particles. For photons leaving a beam splitter these states can be labeled in terms of the output port and the particle number. We denote particle 1 leaving port 3 as $|3_1\rangle$, particle 2 leaving port 4 as $|4_2\rangle$, and so on. Obviously there can be just four distinct product states. Students will have seen that out of linear combinations of these four product states, it is possible to construct four mutually orthogonal states that have a definite symmetry under the exchange of two particles:

$\|3_1\rangle\|3_2\rangle$	(a) Exchange symmetric, both photons exit port 3.
$\frac{1}{\sqrt{2}}[\|3_1\rangle\|4_2\rangle+\|3_2\rangle\|4_1\rangle]$	(b) Exchange symmetric, one photon exits port 3, the other port 4.
$\|4_1\rangle\|4_2\rangle$	(c) Exchange symmetric both photons exit port 4.
$\frac{1}{\sqrt{2}}[\|3_1\rangle\|4_2\rangle-\|3_2\rangle\|4_1\rangle]$	(d) Exchange antisymmetric, one photon exits port 3, the other port 4.

(8)

If we replace output port numbers 3 and 4 with input port numbers 1 and 2, respectively, we obtain the analogous four equations for the input states.

If phase shifters are placed in the arms of the interferometer and the output photons are detected in coincidence, an interference pattern will be observed in the coincidences even though none is observed in the counts of the individual detectors. Greenberger, Horne, and Zeilinger[15] give a good explanation of the essential features of such two-photon interference, and Loudon[16] gives a thorough treatment using the formalism of creation and annihilation operators. We leave for another paper the discussion of our pedagogical approach to presenting this material to our students.

Two-photon coincidences can provide a dramatic example of the quantum effect of indistinguishability. The analysis is also a good application of Feynman's rules for working with probability amplitudes.[1,10] For the setup shown in Fig. 3, there are only two ways that coincidences between D_1 and D_2 can occur. Either each photon reflects at the beam splitter or each photon passes through it. The amplitude for two reflections and two transmissions is *rr* and *tt*, respectively. If the two path lengths through the apparatus are different, then *rr* is physically distinguishable from *tt*, and the probability that there will be counts in both D_1 and D_2 is $|rr|^2+|tt|^2 = 1/2$, the sum of the squares of the amplitudes of the two distinguishable cases. But when the path lengths are made equal, the events *rr* and *tt* become indistinguishable, and the probability is then the square of the sum of the individual amplitudes: $|rr+tt|^2$. For a symmetric beam splitter $r=1$, $t=i$, and $rr+tt=1-1=0$. There is no amplitude for coincidences between D_1 and D_2. The effect is seen experimentally[17] as the appearance of a sharp minimum in the coincidence rate as the interferometer arms are adjusted to be of equal length.

As noted, this result is for incident photons in identical polarization states. When we take into account other polarization states, a new and interesting possibility emerges.

D. Polarization

Because it can have either of two independent modes of polarization, a single photon is itself a two-state system. For example, with appropriate polarizers a photon can be put into a state of vertical polarization, $|V\rangle$, or horizontal polarization, $|H\rangle$. These states constitute a basis, and any general polarization state can be expressed as a linear combination of $|H\rangle$ and $|V\rangle$. An arbitrary two-photon polarization state can then be described in terms of four product states that are the exact analogues of the spatial states described by Eq. (8). These can be obtained from Eq. (8) by replacing port numbers 3 and 4, respectively, with V and H.

A complete description of the two-photon states of a beam splitter can be given in terms of the sixteen products of the four spatial states with the four polarization states. These constitute a complete set of orthogonal states for describing two particles in a beam splitter. Because photons are bosons, the only possible states are linear combinations of the ten states that are symmetric under the exchange of the particle labels. Nine of these states are products of the exchange symmetric spatial states with the exchange symmetric polarization states. None of these states yields a photon in each output channel. For six of the states, this null result is because the spatial part of the state vector is either $|3_1\rangle|3_2\rangle$ or $|4_1\rangle|4_2\rangle$ that is, two photons leaving by the same output port. The other three states have $(1/\sqrt{2})[|3_1\rangle|4_2\rangle$

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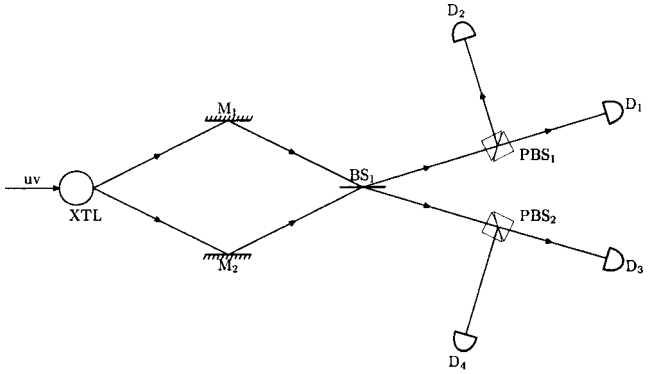

Fig. 5. A Mach–Zehnder interferometer with polarizing beam splitters, PBS_1 and PBS_2, to analyze photons exiting from the beam splitter BS_1.

$+|3_2\rangle|4_1\rangle]$ as a factor, but this factor is zero because, as the previous section showed, the amplitude $rr+tt$ is zero for a 50–50 symmetric beam splitter.

The tenth exchange symmetric state is the product of the antisymmetric spatial state with the antisymmetric polarization state:

$$\left[\frac{1}{\sqrt{2}}(|3_1\rangle|4_2\rangle-|3_2\rangle|4_1\rangle)\right]\left[\frac{1}{\sqrt{2}}(|V_1\rangle|H_2\rangle-|V_2\rangle|H_1\rangle)\right]. \quad (9)$$

Now because of the exchange antisymmetry, the two amplitudes for producing coincidences subtract when the interferometer arms are exactly equal in length, that is, the probability for coincidences is $P_c=|rr-tt|^2=1$. Therefore, if the initial two-photon state contains any antisymmetric spatial part, there will be D_1-D_2 coincidences. This result is possible only if the entering photons have different states of polarization.

Equation (9) is an example of what Schrödinger called an "entangled state," and in our course this will be a good place to begin a discussion of the properties and consequences of such states. In such states photons do not have individual identity, although they are correlated. There is no way to predict the polarization state of a photon in a particular output port, but once it has been measured and found to be, say, V, the photon in the other output port will always be measured to be H, and conversely. This is the situation that Einstein, Podolsky, and Rosen called to our attention nearly 70 years ago,[18] and which Einstein deplored as giving rise to "spooky action at a distance."

We can demonstrate the correlation of the photon states experimentally using polarizing beam splitters. The polarization state of photons exiting from such a device depends on which output port they exit. For example, in Fig. 5 the apparatus can be arranged so that photons coming out upward from PBS_1 are in a $|H\rangle$ state, while those going straight are in the $|V\rangle$ state. Then D_1 will be detecting $|V\rangle$ photons and D_2 will be detecting $|H\rangle$ photons. Suppose further, for the sake of example, that the apparatus is arranged so that D_3 detects $|V\rangle$ photons while D_4 detects $|H\rangle$ photons. Because of the polarization correlation, an input entangled state of the form Eq. (9) would then give rise to coincidences only between D_1 and D_4 or between D_2 and D_3, and never between D_1 and D_3 or between D_2 and D_4. These properties of two-photon states of a beam splitter have been adroitly used to exhibit quantum teleportation.[19,20]

V. CONCLUSIONS

Development of the first experiment is under way, and we are preparing instructional materials to foster student understanding of the experiments. The module on beam splitters described here is ready to be tried in our quantum mechanics course. We have also begun a module on Bell's inequalities that has been used twice in the classroom. Further development of that module is continuing along with the development of modules on the quantum eraser and on two-photon interference.

ACKNOWLEDGMENTS

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[a]Electronic mail: cholbrow@colgate.edu

[1]R. Y. Chiao, P. G. Kwiat, and A. M. Steinberg, "Optical tests of quantum mechanics," in *Advances in Atomic, Molecular, and Optical Physics* (Academic, New York, 1994), Vol. 34, pp. 35–83.

[2]Our Web site is at http://departments.colgate.edu/physics/research/Photon/photon_quantum_mechanics.htm.

[3]D. Griffiths, *Quantum Mechanics* (Prentice–Hall, Englewood Cliffs, NJ, 1995), pp. 154–176. Chapter 3 of this book supplies a nice summary of the concepts and terminology of linear algebra relevant to quantum mechanics (pp. 75–120). It may be too brief for our purposes.

[4]J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge U.P., Cambridge, 1987). See particularly "On the Einstein-Podolsky-Rosen paradox," pp. 14–21, and "Bertlmann's socks and the nature of reality," pp. 139–158.

[5]D. Mermin, "Is the moon there when nobody looks? Reality and quantum theory," Phys. Today **38**(4), 38–47 (1985).

[6]An important long-range goal of our project is to achieve a design that exploits advances in technology to bring the cost of the apparatus within the budgets of colleges.

[7]A significant part of this laboratory experiment will be devoted to introducing students to the statistics of counting.

[8]A. P. French and Edwin F. Taylor, *Introduction to Quantum Physics* (Chapman and Hall, London, 1974), pp. 231–278.

[9]H. J. Lipkin, "Polarized photons for pedestrians," in *Quantum Mechanics: New Approaches to Selected Topics* (North-Holland, New York, 1973), pp. 2–29.

[10]R. P. Feynman, R. B. Leighton, and M. L. Sands, *The Feynman Lectures on Physics* (Addison–Wesley, Reading, MA, 1963), Vol. 3, Chaps. 5 and 6. Feynman develops quantum mechanics largely in terms of two-state systems. We will make extensive use of his material.

[11]R. P. Feynman, *QED, the Strange Theory of Light and Matter* (Princeton U.P., Princeton, 1985).

[12]D. F. Styer, *The Strange World of Quantum Mechanics* (Cambridge U.P., New York, 2000).

[13]A. Zeilinger, "General properties of lossless beam splitters in interferometry," Am. J. Phys. **49**, 882–883 (1981). This article expresses the general 2×2 unitary matrix in terms of Pauli spin matrices. This approach would be particularly useful for further familiarizing students in a quantum mechanics course with these important mathematical tools.

[14]The exit probabilities are independent of whether the transmitted beam leads or lags the reflected beam by $\pi/2$ rad. In other words, a matrix **R** in which $t=-i$ is equivalent to one with $t=i$.

[15]D. M. Greenberger, M. A. Horne, and A. Zeilinger, "Multiparticle interferometry and the superposition principle," Phys. Today **46**(8), 22–29 (1993). This article is an excellent source of explanations of the kinds of quantum effects that our project is intended to display.

[16]R. Loudon, "Spatially-localized two-particle interference at a beam split-

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EXISTENCE OF ATOMS

"I Don't believe that atoms exist!"

This blunt declaration of disbelief came, in fact, in January 1897 at a meeting of the Imperial Academy of Sciences in Vienna. The skeptic was Ernst Mach, not quite 60 years old, who had been for many years a professor of physics at the University of Prague and who was now a professor of history and philosophy of science in Vienna. He pronounced his uncompromising opinion in the discussion following a lecture delivered by Ludwig Boltzmann, a theoretical physicist. Boltzmann, a few years younger than Mach, had likewise recently returned to Vienna after many years at other universities in Austria and Germany. He was an unabashed believer in the atomic hypothesis—indeed, his life's work had centered on that single theme.

Davie Lindley, *Boltzmann's Atom* (The Free Press, New York, NY, 2001), page 1 of the introduction.

Submitted by Louise Moll Claver.

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